Advanced Systems Lab

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Lecture: Optimization for Instruction-Level Parallelism

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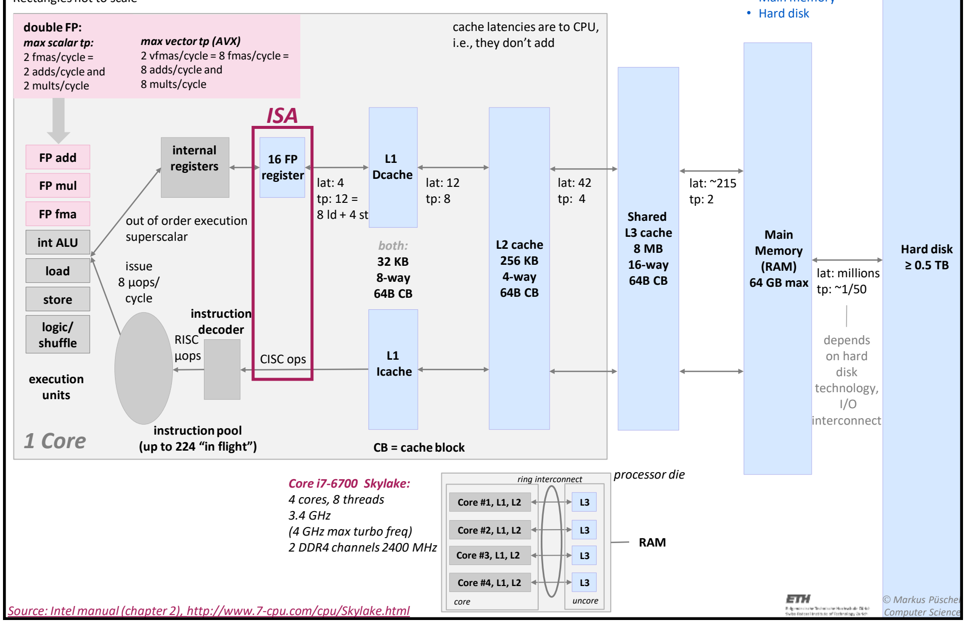
Abstracted Microarchitecture: Example Core i7 Skylake (2015)

Throughput (tp) is measured in doubles/cycle. For example: 4. *Numbers are for loading into registers.*
Latency (lat) is measured in cycles
1 double floating point (FP) = 8 bytes
fma = fused multiply-add
Rectangles not to scale

Memory hierarchy:

- Registers
- L1 cache
- L2 cache
- L3 cache
- Main memory
- Hard disk



Source: Intel manual (chapter 2), http://www.7-cpu.com/cpu/Skylake.html

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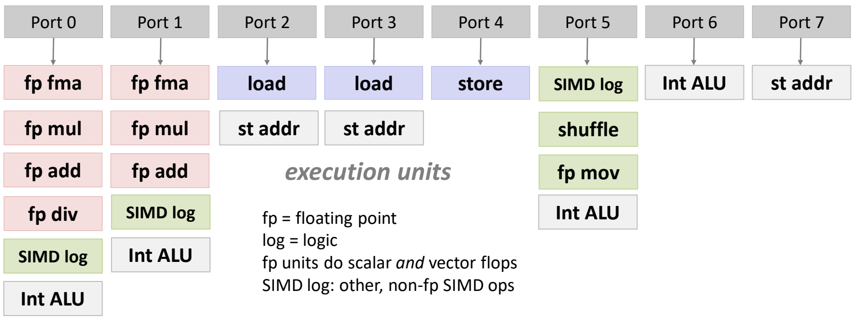
Execution Units and Ports (Skylake)



fp = floating point
log = logic
fp units do scalar *and* vector flops
SIMD log: other, non-fp SIMD ops

Execution Unit (fp)	Latency [cycles]	Throughput [ops/cycle]	1/Throughput [cycles/issue]
fma	4	2	0.5
mul	4	2	0.5
add	4	2	0.5
div (scalar) div (4-way)	14 14	1/4 1/8	4 8

- Every port can issue one instruction/cycle
- ***Intel calls 1/throughput the throughput!***
- Same exec units for scalar and vector flops
- Same latency/throughput for scalar (one double) and AVX vector (four doubles) flops, except for div
- Check Agner Fog's tables (pp. 278)

Source: Intel manual (Table C-8. 256-bit AVX Instructions, Table 2-1. Dispatch Port and Execution Stacks of the Skylake Microarchitecture, Figure 2-1. CPU Core Pipeline Functionality of the Skylake Microarchitecture)

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How To Make Code Faster?

It depends! First high-level approaches:

Memory bound: Reduce data movement

- *Reduce cache misses*
- *Compress data*

Compute bound: Keep floating point units busy

- *Reduce cache misses, register spills*
- *Instruction level parallelism (ILP)*
- *Vectorization*

Next: Optimizing for ILP (an example)

Chapter 5 in Computer Systems: A Programmer's Perspective, 2nd edition, Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010
Part of these slides are adapted from the course associated with this book

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Superscalar Processor

Definition: A superscalar processor can issue and execute *multiple instructions in one cycle*. The instructions are retrieved from a sequential instruction stream and are usually scheduled dynamically.

Benefit: Superscalar processors can take advantage of *instruction level parallelism (ILP)* that many programs have.

Most desktop/laptop CPUs since about 1998 are superscalar

Intel: since Pentium Pro

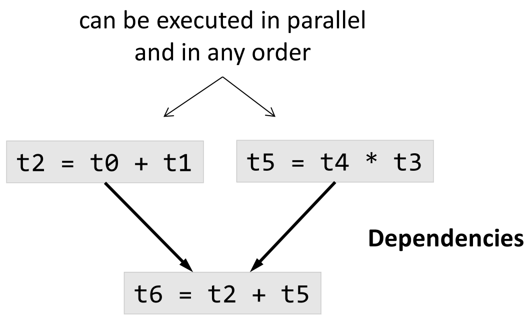
Simple embedded processors are usually not superscalar

Deep pipelines also require ILP (explained today)

ILP

Code

```
t2 = t0 + t1
t5 = t4 * t3
t6 = t2 + t5
```



Hard Bounds: Coffee Lake and Haswell

Coffee Lake (2017, Skylake family)

	latency	1/throughput
FP Add	4	0.5
FP Mul	4	0.5
Int Add	1	0.5
Int Mul	3	1

blackboard (FP Add, FP Mul)

More precisely (Int Add):
- tp = 2 if one operand is from mem
- tp = 4 if all operands are in register

Haswell (2013)

	latency	1/throughput
FP Add	3	1
FP Mul	5	0.5
Int Add	1	0.5
Int Mul	3	1

Only the ALU at Port 1 does Int Mults

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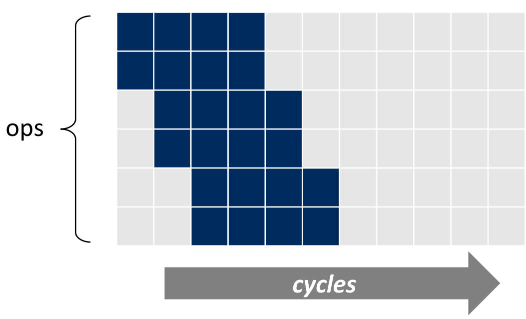
Coffee Lake

	latency	1/throughput
FP Mul	4	0.5



Throughput: 2/cycle

How many cycles at least for n mults?

- *ceil(n/2) (considering only throughput)*
- *ceil(n/2) + 3 (considering latency and throughput)*

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Example Computation: Reduction

```
void reduce(vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(v);
  data_t *d  = get_vec_start(v);
  data_t t   = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

d[0] OP d[1] OP d[2] OP … OP d[length-1]

data_t: double or int

OP: + or *

IDENT: 0 or 1

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Runtime of Reduce (Coffee Lake)

```
void reduce(vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(v);
  data_t *d  = get_vec_start(v);
  data_t t   = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

Measured cycles per OP

Method	Int (add/mult)		Float (add/mult)	
reduce	1.29	2.95	3.91	3.91
bound	0.5	1.0	0.5	0.5

Questions:

- *Explain red row*
- *Explain gray row*

This and all following measurements: gcc -O3 -mavx2 -fno-tree-vectorize

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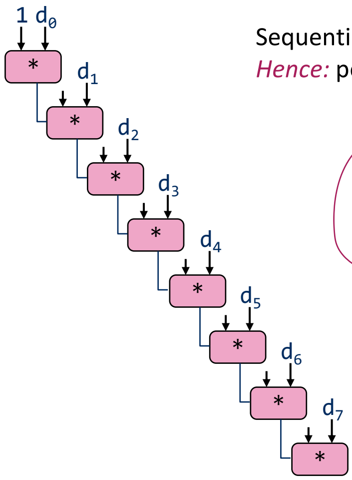
Reduce = Serial Computation (here: *)



Sequential dependence = no ILP!
Hence: performance determined by latency of OP!

Method	Int (add/mult)		Float (add/mult)	
reduce	1.29	2.95	3.91	3.91
bound	0.5	1.0	0.5	0.5

Loop Unrolling

```
void unroll2(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit  = length-1;
    data_t *d  = get_vec_start(v);
    data_t x   = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i += 2)
        x = (x OP d[i]) OP d[i+1];
    /* Finish any remaining elements */
    for (; i < length; i++)
        x = x OP d[i];
    *dest = x;
}
```

Perform 2x more useful work per iteration

How does the runtime change?

Effect of Loop Unrolling

Method	Int (add/mult)		Float (add/mult)	
combine4	1.29	2.95	3.91	3.91
unroll2	1.0	2.93	3.90	3.91
bound	0.5	1.0	0.5	0.5

Helps integer sum a bit

Others don't improve. *Why?*

- *Still sequential dependency*

```
x = (x OP d[i]) OP d[i+1];
```


Loop Unrolling with Separate Accumulators

```
void unroll2_sa(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit  = length-1;
    data_t *d  = get_vec_start(v);
    data_t x0  = IDENT;
    data_t x1  = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i += 2) {
       x0 = x0 OP d[i];
       x1 = x1 OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++)
        x0 = x0 OP d[i];
    *dest = x0 OP x1;
}
```

Effect on runtime?

Can this change the result of the computation?

Floating point: yes!

Effect of Separate Accumulators

Method	Int (add/mult)		Float (add/mult)	
combine4	1.29	2.95	3.91	3.91
unroll2	1.0	2.93	3.90	3.91
unroll2-sa	0.8	1.49	1.96	1.97
bound	0.5	1.0	0.5	0.5

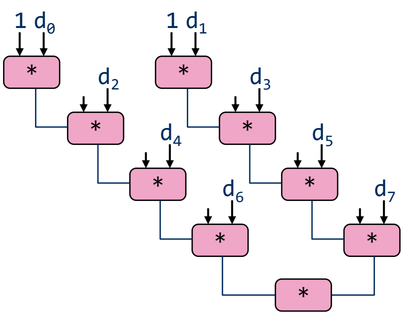
Almost exact 2x speedup (over unroll2) for Int *, FP +, FP *

- *Breaks sequential dependency*

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```


Separate Accumulators

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```



What changed:

- Two independent "streams" of operations

Overall Performance

- N elements, D cycles latency/op
- Should be (N/2+1)*D cycles:
  cycles per OP ≈ D/2

What Now?

Unrolling & Accumulating

Idea

- *Use K accumulators*
- *Increase K until best performance reached*
- *Need to unroll by L, K divides L*

Limitations

- *Diminishing returns:*
  Cannot go beyond throughput limitations of execution units
- *Some overhead for short lengths: Finish off iterations sequentially*

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Unrolling & Accumulating: FP *

Coffee Lake: FP multiplication

- *1/Throughput = cycles/issue = 0.5*
- *Throughput = 2*
- *Latency = 4*

Accumulators

FP64 *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	3.91	3.91		3.91		3.91		
2		1.97		1.97		1.96		
3			1.32		1.32			
4				1.00		1.0		
6					0.70			0.70
8						0.56		
10							0.54	
12								0.54

Why 8?

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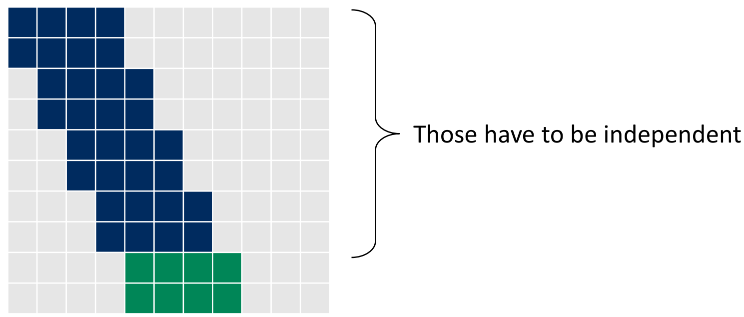
Why 8?



Based on this insight: K = #accumulators
= ceil(latency / cycles per issue)
= ceil(latency * throughput)

Here (FP Mult): K = ceil(4 / 0.5) = ceil(4 * 2) = 8

Unrolling & Accumulating: FP +

Coffee Lake: FP addition

- *Throughput = 2*
- *Latency = 4*

FP64 +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	3.91	3.90		3.90		3.90		
2		1.96		1.96		1.96		
3			1.32		1.32			
4				1.00		1.00		
6					0.70			0.70
8						0.56		
10							0.54	
12								0.54

Accumulators

Unrolling & Accumulating: Int *

Coffee Lake: Int multiplication

- *Throughput = 1*
- *Latency = 3*

Accumulators

Int *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	2.94	2.94		2.93		2.93		
2		1.49		1.49		1.49		
3			1.32		1.32			
4				1.01		1.01		
6					1.01			1.00
8						1.01		
10							1.01	
12								1.01

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Unrolling & Accumulating: Int +

Coffee Lake: Int multiplication

- *Throughput = 2*
- *Latency = 1*

Accumulators

Int +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	1.29	1.00		1.00		1.00		
2		0.80		0.58		0.52		
3			0.69		0.52			
4				0.57		0.52		
6					0.52			0.52
8						0.52		
10							0.52	
12								0.52

Interesting question: what exactly happens here?

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Coffee Lake vs. Haswell: FP +

FP64 +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	3.91	3.90		3.90		3.90		
2		1.96		1.96		1.96		
3			1.32		1.32			
4				1.00		1.00		
6					0.70			0.70
8						0.56		
10							0.54	
12								0.54

Coffee Lake:
Latency = 4
Throughput = 2

FP64 +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	2.95	2.95		2.95		2.95		
2		1.49		1.49		1.49		
3			1.00		1.00			
4				1.01		1.01		
6					1.01			1.01
8						1.00		
10							1.01	
12								1.01

Haswell:
Latency = 3
Throughput = 1

Says something about porting processor-tuned code

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Apple M1: FP +

FP64 +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	2.87	2.83		2.75		2.75		
2		1.38		1.39		1.26		
3			0.93		0.93			
4				0.71		0.71		
6					0.49			0.49
8						0.38		
10							0.32	
12								0.31

Firestorm:
Latency = 3
Throughput = 4

about 9x speedup!

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Summary (ILP)

Deep pipelines and multiple ports require ILP for good performance

ILP may have to be made explicit in program

Potential blockers for compilers
- *Reassociation changes result (floating point)*
- *Too many choices, no good way of deciding*

Unrolling
- *By itself does usually nothing (branch prediction works usually well)*
- *But may be needed to enable additional transformations (here: reassociation)*

How to program this example?
- *Solution 1: program generator generates alternatives and picks best*
- *Solution 2: use model based on latency and throughput*

We have seen how to obtain runtime bounds based on throughput and runtime estimates considering latency. Can be done on arbitrary computation DAGs.