Advanced Systems Lab

Spring 2024

Lecture: Roofline model

Instructor: Markus Püschel

TA: Tommaso Pegolotti, several more

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

1

Operational Intensity Again

Definition: Given a program P, assume cold (empty) cache

Operational intensity: $I(n) = \frac{W(n)}{Q(n)}$

- W(n): #flops (input size n)
- Q(n): #bytes transferred cache ↔ memory (for input size n)

Asymptotic bounds on I(n)

Vector sum: y = x + y	O(1)	
Matrix-vector product: y = Ax	O(1)	
Fast Fourier transform	O(log(n))	O(log(γ)) ← (not explained)
Matrix-matrix product: C = AB + C	O(n)	→ O(sqrt(γ))

Cache lecture
γ = size LLC (last level cache)
Known to be optimal

2

2

Compute/Memory Bound

A function/piece of code is:

- ***Compute bound*** *if it has high operational intensity*
- ***Memory bound*** *if it has low operational intensity*

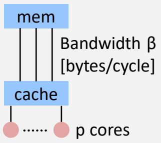
The roofline model makes this more precise

3

3

Roofline model/plot (Williams et al. 2008)

Platform model



← carefully measured
- raw bandwidth from manual unattainable (maybe 60% is)
- Stream benchmark may be conservative

Each with peak performance π [flops/cycle]

Algorithm/program model (n is the input size)

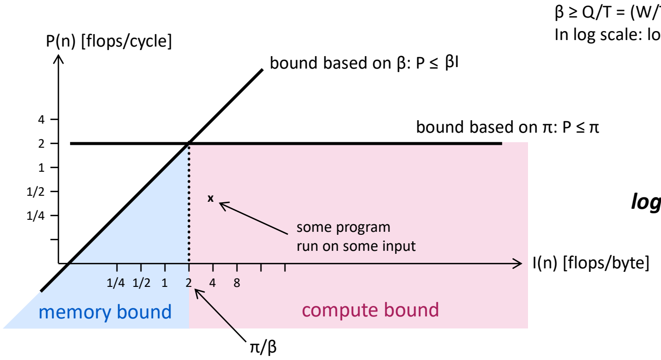
Work	W(n) [flops]
Data movement:	Q(n) [bytes]
(mem ↔ cache)	
Runtime:	T(n) [cycles]

Derived:

Operational intensity	I(n) = W(n)/Q(n) [flops/byte]
Performance:	P(n) = W(n)/T(n) [flops/cycle]

Example: one core, $\pi = 2$, $\beta = 1$, no SIMD

Bound based on β:
$\beta \geq Q/T = (W/T)/(W/Q) = P/I$
In log scale: $\log_2(P) \leq \log_2(\beta) + \log_2(I)$

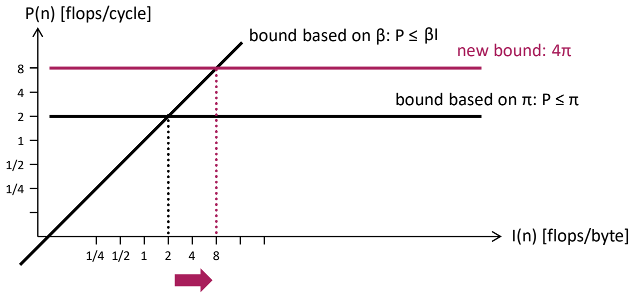


log-log scale!

4

Roofline Plots

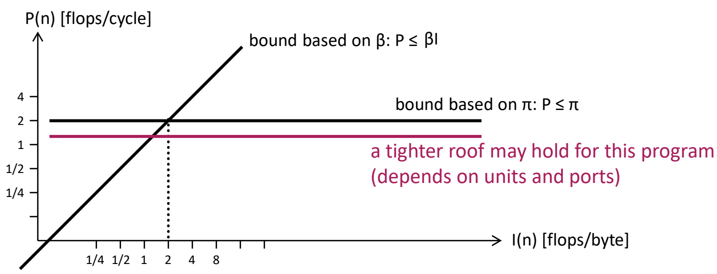
What happens if we introduce 4-way SIMD?



If β does not change:
more programs become memory bound

Roofline Plots

What if a program has an uneven mix of operations (e.g., 20% mults and 80% adds)?



Roofline Measurements

Tool developed in our group (code may need an update)
(G. Ofenbeck, R. Steinmann, V. Caparros-Cabezas, D. Spampinato)
http://www.spiral.net/software/roofline.html

Example plots follow

Estimate operational intensity I = W/Q (cold cache):

- *daxpy:* *y = αx+y* W = 2n Q = 3n doubles = 24n bytes I = 1/12
- *dgemv:* *y = Ax + y* W = $2n^2$ Q ≈ n^2 doubles = $8n^2$ bytes I ≈ 1/4
- *dgemm:* *C = AB + C* W = $2n^3$ Q ≥ $4n^2$ doubles = $32n^2$ bytes I ≤ n/16
- *FFT*

Note:

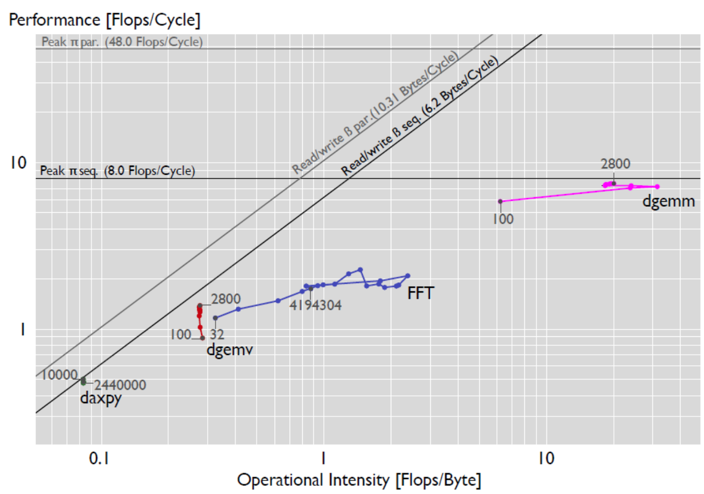
- *For daxpy and dgemv, Q is determined by compulsory misses.*
- *For dgemm, more misses than compulsory misses occur for larger sizes.*
  If $3n^2 \leq \gamma$ (cache size), equality should hold above.

7

7

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Intel MKL, ***sequential***
Cold cache



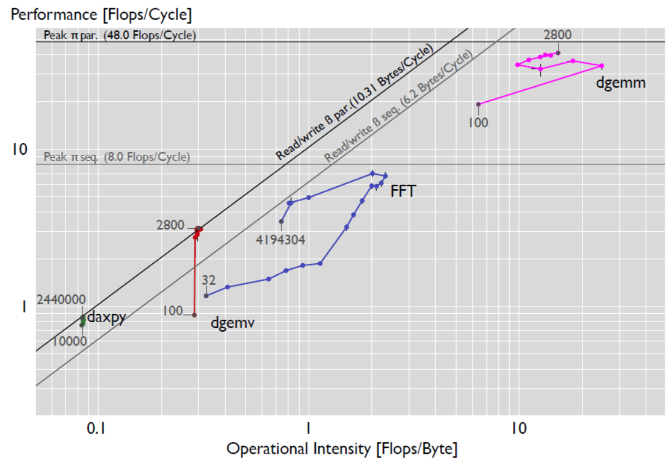
What happens when we go to parallel code?

8

8

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
*Code: Intel MKL, **parallel***
Cold cache



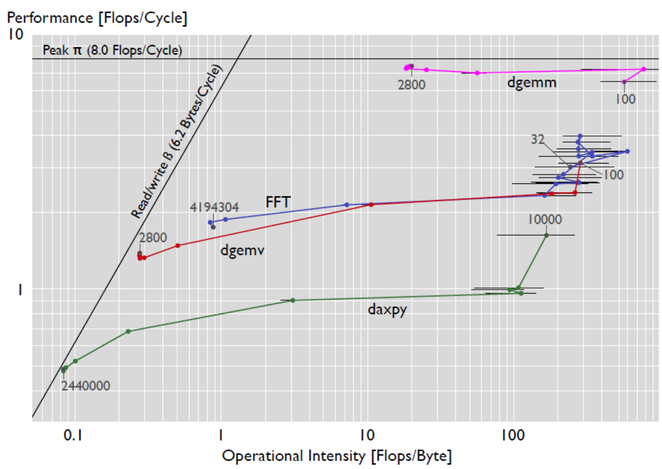
What happens when we measure with warm cache?

9

9

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
*Code: Intel MKL, **sequential***
Warm cache

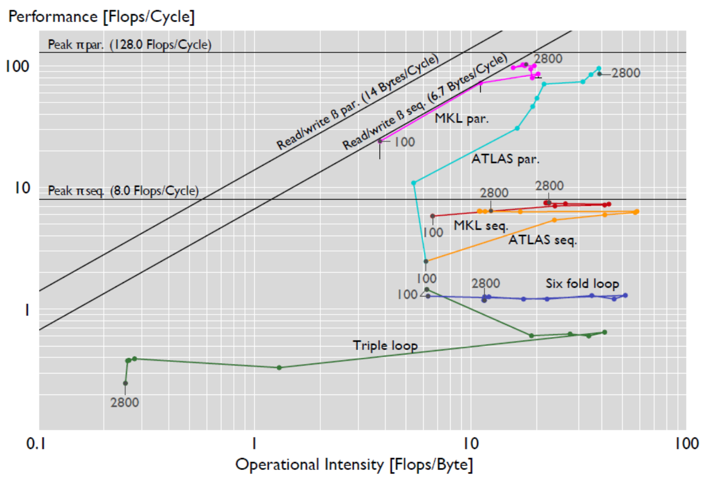


10

10

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Various MMM
Cold cache



MMM: Different implementations

11

11

Cache-Aware Roofline Model (CARM)

Changes the definition of Q to count all loads from the memory hierarchy, i.e., from memory and all levels of caches.

Shows throughput bounds for all levels of caches

Intel Advisor:

- *Uses CARM as standard*
- *Provides the standard model, called MLR, but has to be used properly, e.g., you have to ensure cold cache and not include initialization*
- *Examples: next slides*

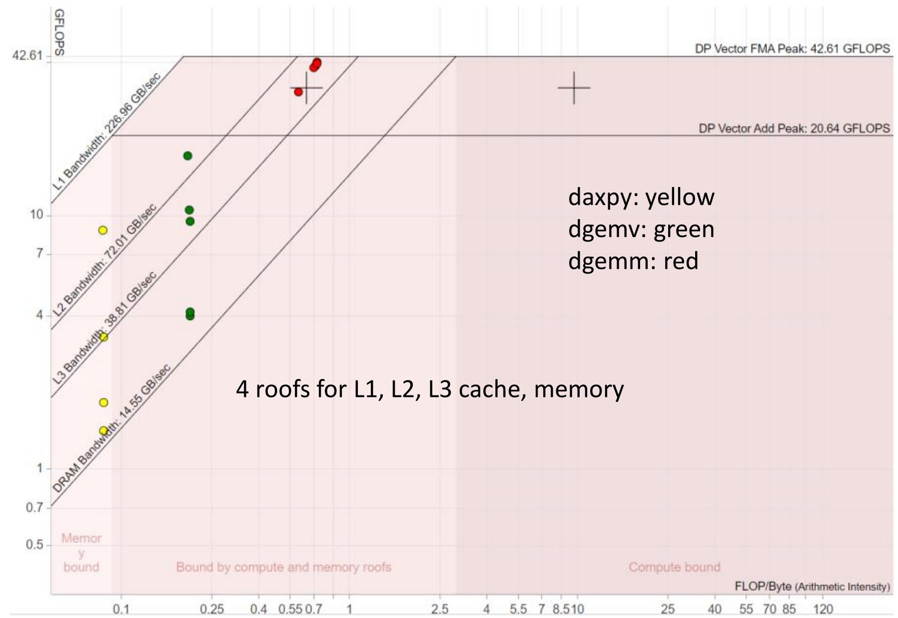
In your project:

- *Do measurements yourself, or*
- *Use Intel Advisor but choose MLR and use properly to reflect the standard roofline model, and explain in report how you did it*

A. Ilic, L. Sousa, Cache-aware Roofline model: Upgrading the loft, IEEE Computer Architecture Letters, 13(1), pp. 21–24, 2014

12

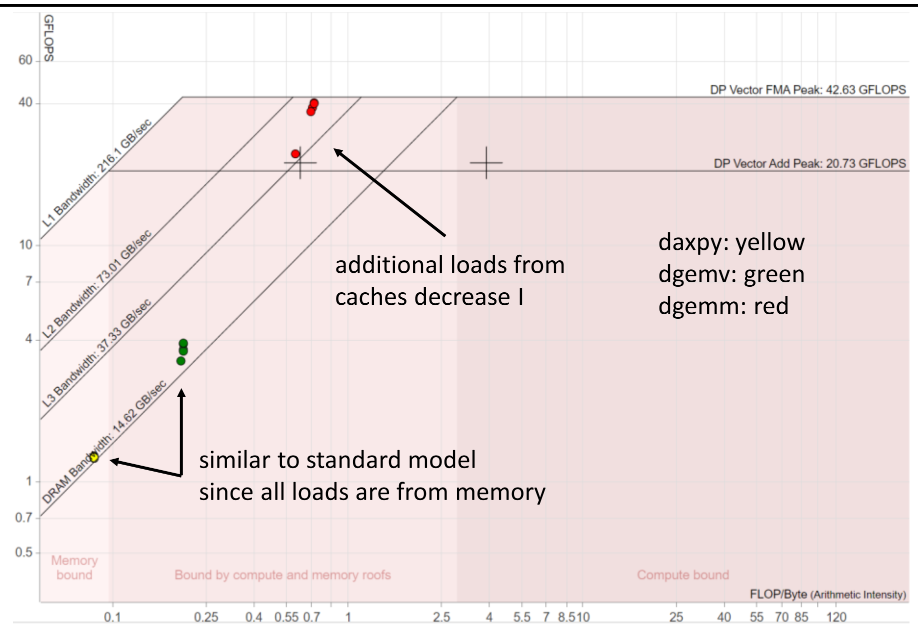
12



CARM, warm cache, including initialization
not good

13

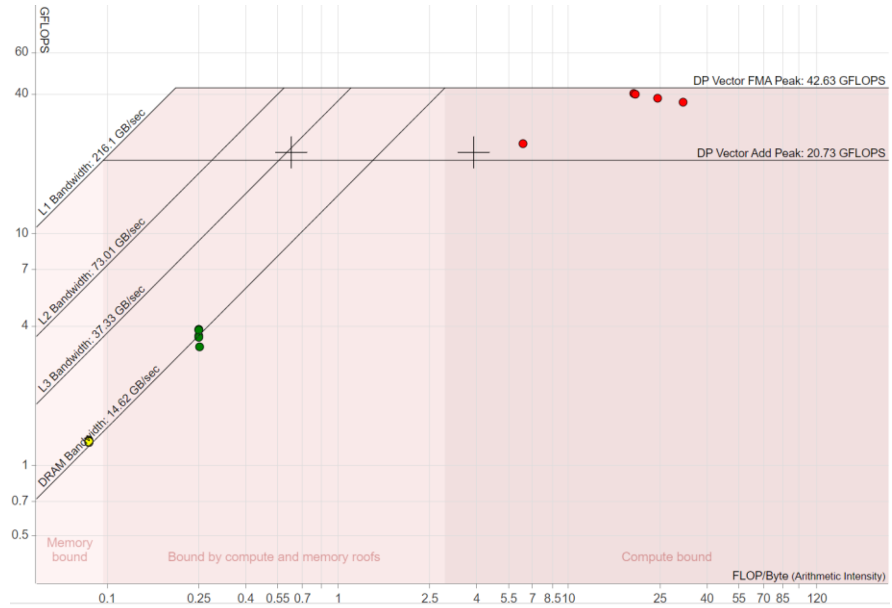
13



CARM, cold cache, excluding initialization
don't use

14

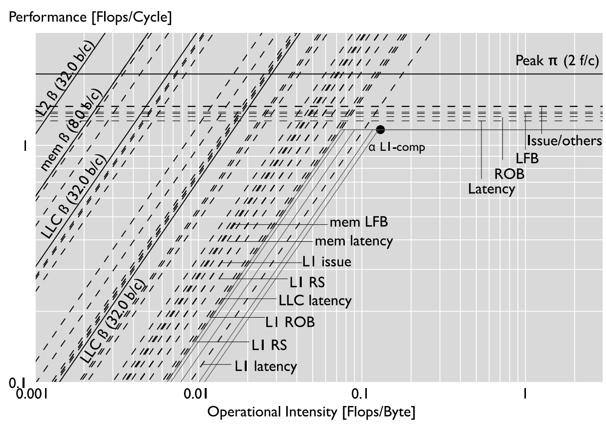
14



MLR, cold cache, excluding initialization
standard roofline model, done well

15

Generalized Roofline Model



Website and tool: https://acl.inf.ethz.ch/research/ERM/

Caparrós Cabezas and P. **Extending the Roofline Model: Bottleneck Analysis with Microarchitectural Constraints,** Proc. IISWC, pp. 222-231, 2014

16

Summary

Roofline plots distinguish between memory and compute bound

Can be instantiated for different scenarios (e.g., computation on GPU, data in CPU memory)

Can be used on for back-of-the-envelope computations on paper

Measurements difficult (performance counters) but doable

Easier variant just consider reads: Q_{read}, β_{read}

Interesting insights: *use in your project, but if so, do properly!*

17