Advanced Systems Lab

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Lecture: Cost analysis and performance

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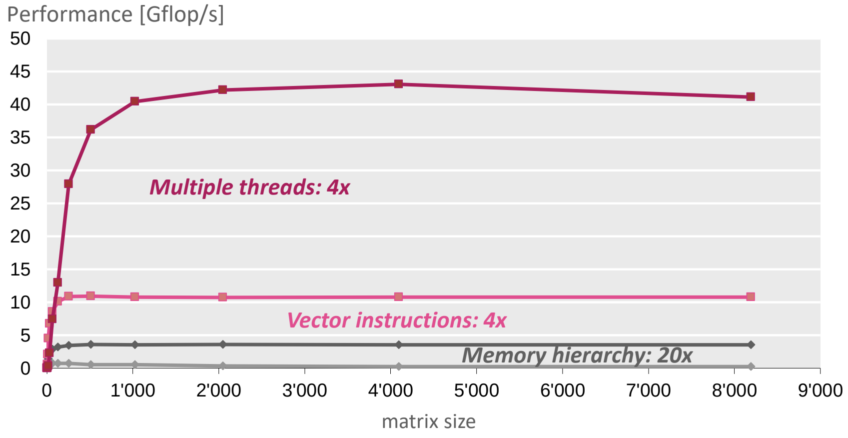
Organization

Team and research project: Deadline: *March 11th*

If you need team: fastcode-forum@lists.inf.ethz.ch

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



- Compiler doesn't do the job
- Doing by hand: ***nightmare***

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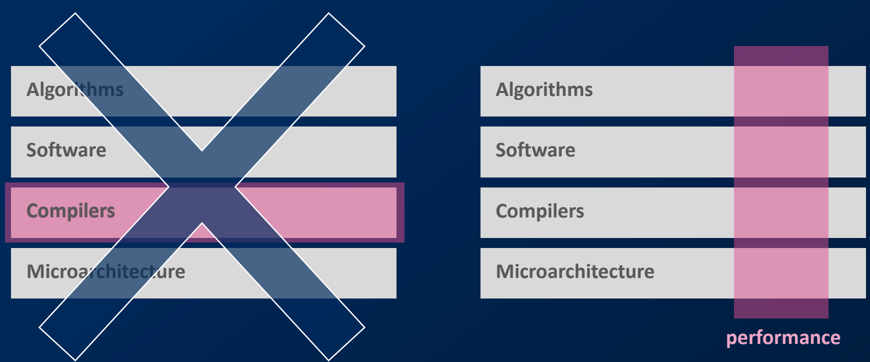

Performance is different than other software quality features

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Today

Asymptotic analysis

Cost analysis and performance

Standard book: Introduction to Algorithms (2nd edition), Corman, Leiserson, Rivest, Stein, McGraw Hill 2001)

Asymptotic Analysis of Algorithms

Analysis for

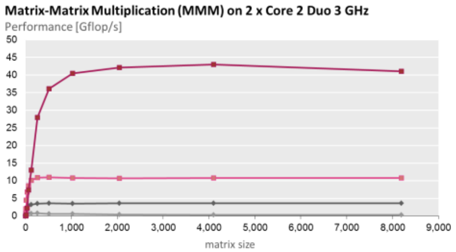
- *Runtime*
- *Space (= memory footprint)*
- *Data movement (e.g., between cache and memory)*

Example MMM: C = A*B + C,   A,B,C are all n x n

- *Runtime:* $O(n^3)$
- *Space:* $O(n^2)$

Valid?

Is asymptotic analysis still valid given this?



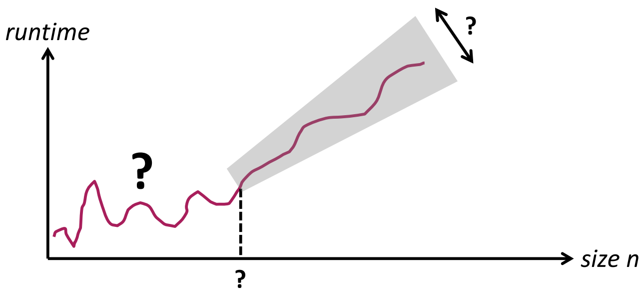
All algorithms are $O(n^3)$ when counting flops.

What happens to asymptotics if I take memory accesses into account?
No problem: O(f(n)) flops means at most O(f(n)) memory accesses

What happens if I take vectorization/parallelization into account?
More parameters needed: E.g., $O(n^3/p)$ on p processors

Asymptotic Analysis: Limitations

Θ(f(n)) describes only the *eventual trend* of the runtime



Constants matter

- *Not clear when "eventual" starts*
- *n^2 is likely better than $1000n^2$*
- *$10000000000n$ is likely worse than n^2*

Cost Analysis for Numerical Problems

Goal: determine exact “cost” of an algorithm

Cost = number of relevant operations

Formally: define *cost measure* C(n). Examples:

- *Counting adds and mults separately: C(n) = (adds(n), mults(n))*
- *Counting adds, mults, divs separately: C(n) = (adds(n), mults(n), divs(n))*
- *Counting all flops together: C(n) = flops(n)*

This course: focus on floating point operations

The cost measure usually counts *only the operations that constitute the mathematical algorithm* (e.g., as written on paper) and not operations that arise due to its mapping on a computer (e.g., index computations, data movement).

Example: next slide

Example

```
/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;

    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k]*b[k*n + j];
}
```

Asymptotic runtime

- $O(n^3)$

Cost measure?

- $C(n) = (fladds(n), flmults(n)) = (n^3, n^3)$
- $C(n) = flops(n) = 2n^3$

Cost Analysis: How To Do

Define suitable cost measure

Count in algorithm or code

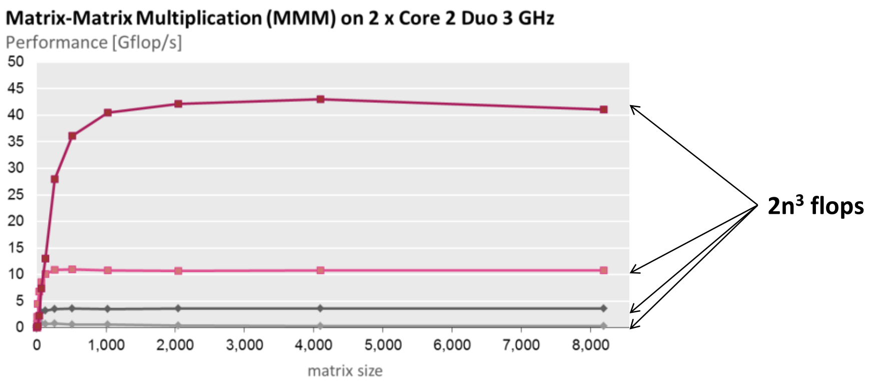
- *Recursive function: solve recurrence*

Instrument code

Use performance counters

- *Intel PCM*
- *Intel Vtune*
- *Perfmon (open source)*
- *Counters for floating points are recently less and less available*

Remember: Even Exact Cost ≠ Runtime

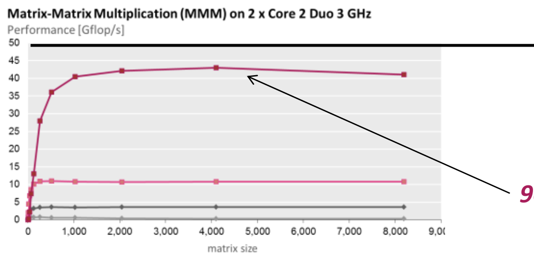


Why Cost Analysis?

Enables performance analysis:

$$\text{performance} = \frac{\text{cost}}{\text{runtime}}$$ *[flops/cycle] or [flops/sec]*

Upper bound through machine's peak performance



Peak performance of this computer

90 % of peak performance

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Example

```
/* Matrix-vector multiplication y = Ax + y */
void mmm(double *A, double *x, double *y, int n) {
    int i, j, k;

    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            y[i] += A[i*n + j]*x[j];
}
```

Flops? For n = 10?

- *$2n^2$, 200*

Performance for n = 10 if runs in 400 cycles

- *0.5 flops/cycle*

Assume peak performance: 2 flops/cycle
percentage peak?

- *25%*

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Summary

Asymptotic runtime gives only an idea of the runtime *trend*

Exact number of operations (cost):

- *Also no good indicator of runtime*
- *But enables performance analysis*
- *Upper bound on performance through computer's peak performance = lower bound on achievable runtime*

Always measure performance (if possible)

- *Gives idea of efficiency*
- *Gives percentage of peak*

In the project, a performance plot is one of the first milestones