Advanced Systems Lab

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Lecture: Roofline model

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Operational Intensity Again

Definition: Given a program P, assume cold (empty) cache

Operational intensity: $I(n) = \frac{W(n)}{Q(n)}$

- $W(n)$: #flops (input size n)
- $Q(n)$: #bytes transferred cache ↔ memory (for input size n)

Asymptotic bounds on I(n)

Vector sum: y = x + y	O(1)	
Matrix-vector product: y = Ax	O(1)	
Fast Fourier transform	O(log(n))	O(log(γ)) ← (not explained)
Matrix-matrix product: C = AB + C	O(n)	→ O(sqrt(γ))

Cache lecture
γ = size LLC (last level cache)
Known to be optimal

Compute/Memory Bound

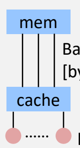
A function/piece of code is:

- ***Compute bound*** *if it has high operational intensity*
- ***Memory bound*** *if it has low operational intensity*

The roofline model makes this more precise

Roofline model/plot (Williams et al. 2008)

Platform model



Bandwidth β [bytes/cycle] ← carefully measured
- raw bandwidth from manual unattainable (maybe 60% is)
- Stream benchmark may be conservative

p cores

Each with peak performance π [flops/cycle]

Algorithm/program model (n is the input size)

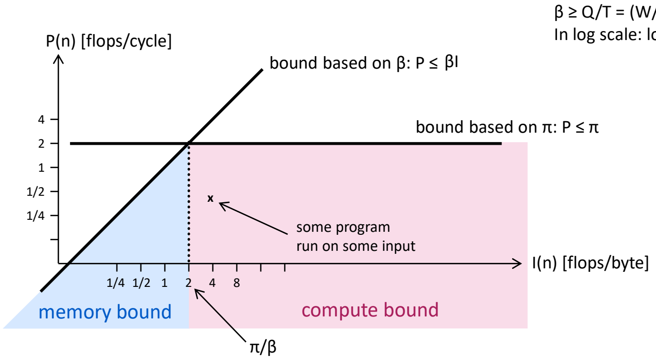
Work	W(n) [flops]
Data movement:	Q(n) [bytes]
(mem ↔ cache)	
Runtime:	T(n) [cycles]

Derived:

Operational intensity	I(n) = W(n)/Q(n) [flops/byte]
Performance:	P(n) = W(n)/T(n) [flops/cycle]

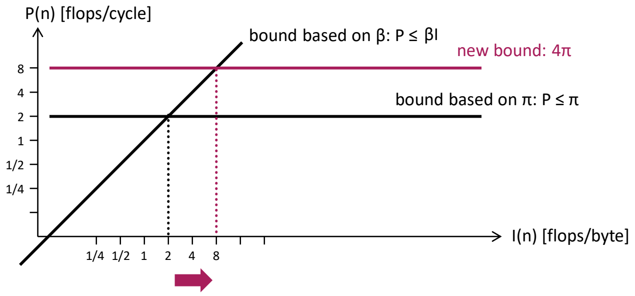
Example: one core, $\pi = 2$, $\beta = 1$, no SIMD

Bound based on β:
$\beta \geq Q/T = (W/T)/(W/Q) = P/I$
In log scale: $\log_2(P) \leq \log_2(\beta) + \log_2(I)$



Roofline Plots

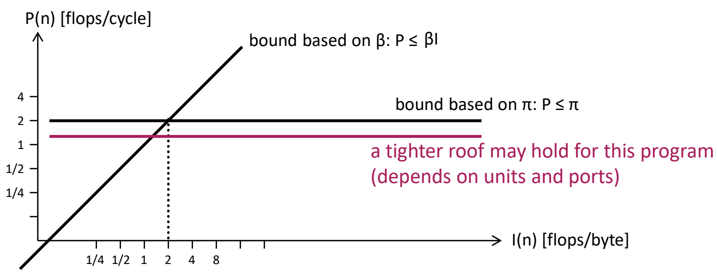
What happens if we introduce 4-way SIMD?



If β does not change:
more programs become memory bound

Roofline Plots

What if a program has an uneven mix of operations (e.g., 20% mults and 80% adds)?



Roofline Measurements

Tool developed in our group (code may need an update)
(G. Ofenbeck, R. Steinmann, V. Caparros-Cabezas, D. Spampinato)
http://www.spiral.net/software/roofline.html

Example plots follow

Estimate operational intensity I = W/Q (cold cache):

- *daxpy:* *y = αx+y* W = 2n Q = 3n doubles = 24n bytes I = 1/12
- *dgemv:* *y = Ax + y* W = $2n^2$ Q ≈ n^2 doubles = $8n^2$ bytes I ≈ 1/4
- *dgemm:* *C = AB + C* W = $2n^3$ Q ≥ $4n^2$ doubles = $32n^2$ bytes I ≤ n/16
- *FFT*

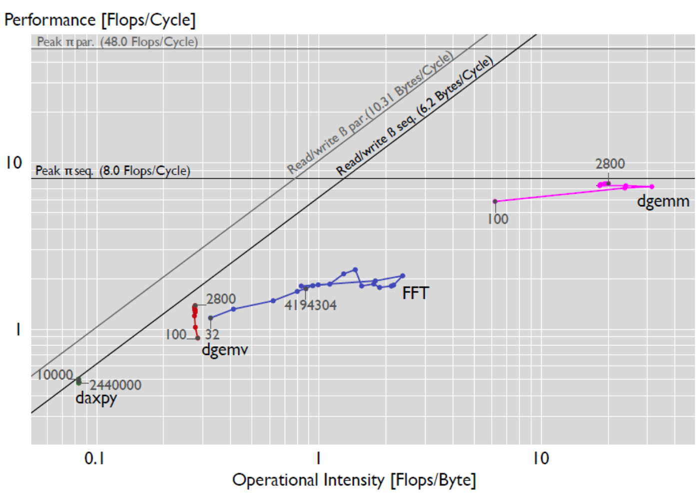
Note:

- *For daxpy and dgemv, Q is determined by compulsory misses.*
- *For dgemm, more misses than compulsory misses occur for larger sizes. If $3n^2 \le \gamma$ (cache size), equality should hold above.*

7

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
*Code: Intel MKL, **sequential***
Cold cache

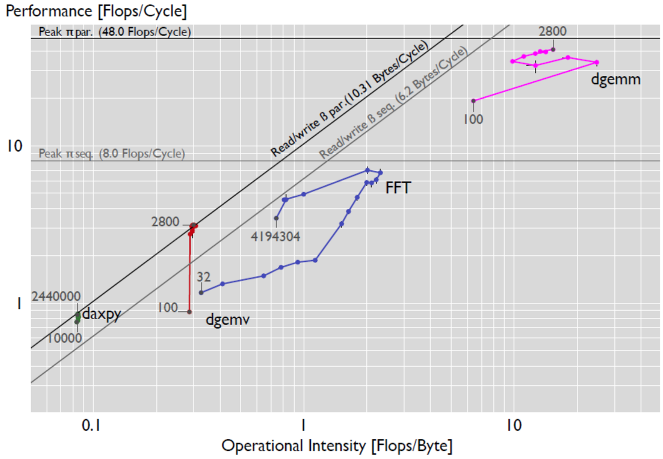


What happens when we go to parallel code?

8

Roofline Measurements

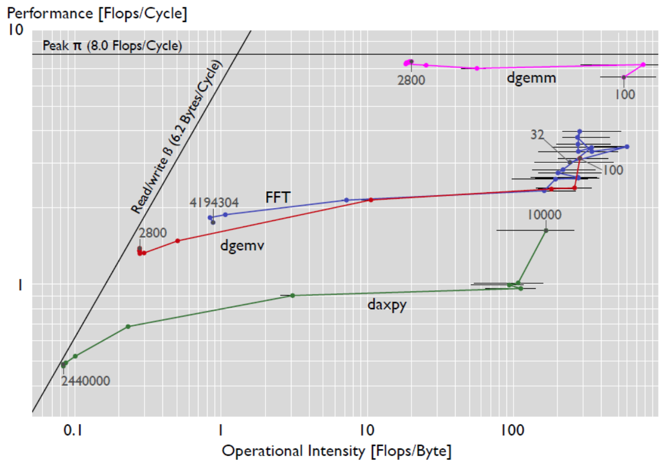
Core i7 Sandy Bridge, 6 cores
*Code: Intel MKL, **parallel***
Cold cache



What happens when we measure with warm cache?

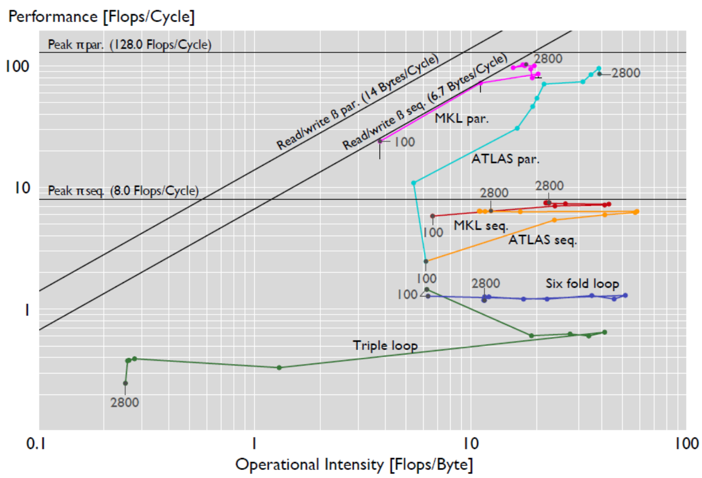
Roofline Measurements

Core i7 Sandy Bridge, 6 cores
*Code: Intel MKL, **sequential***
Warm cache



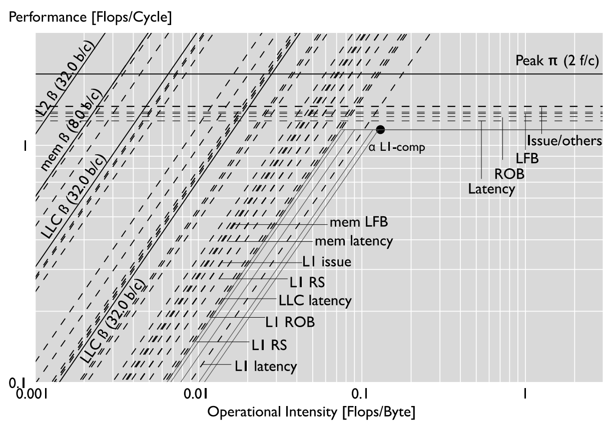
Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Various MMM
Cold cache



MMM: Different implementations

Generalized Roofline Model



Website and tool: https://acl.inf.ethz.ch/research/ERM/

Caparrós Cabezas and P. **Extending the Roofline Model: Bottleneck Analysis with Microarchitectural Constraints,** Proc. IISWC, pp. 222-231, 2014

Summary

Roofline plots distinguish between memory and compute bound

Can be used on for back-of-the-envelope computations on paper

Measurements difficult (performance counters) but doable

Interesting insights: *use in your project!*