Advanced Systems Lab

Spring 2021

Lecture: Memory hierarchy, locality, caches

Instructor: Markus Püschel, Ce Zhang

TA: Joao Rivera, several more

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Organization

Temporal and spatial locality

Memory hierarchy

Caches

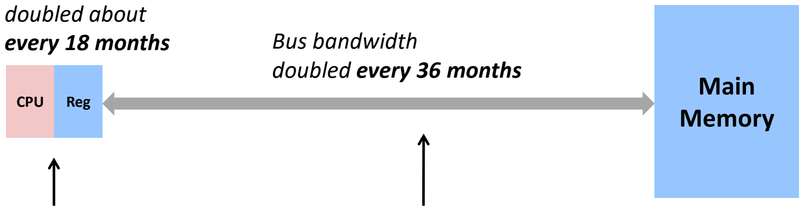
Chapter 5 in Computer Systems: A Programmer's Perspective, 2nd edition, Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010

Part of these slides are adapted from the course associated with this book

Problem: Processor-Memory Bottleneck

Processor performance doubled about ***every 18 months***

Bus bandwidth doubled ***every 36 months***

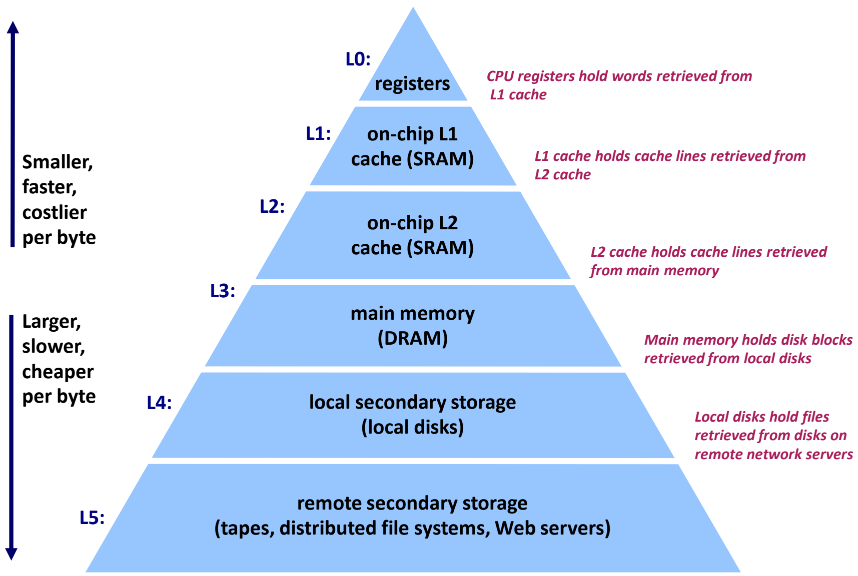


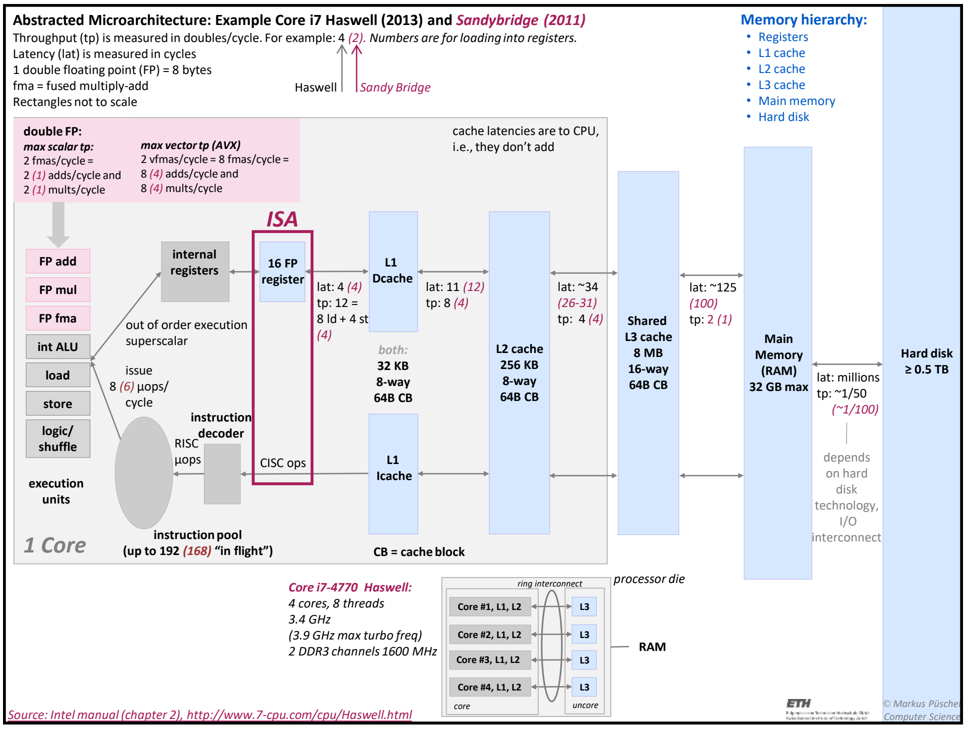
Core i7 Haswell:
Peak performance:
2 AVX three operand (FMA) ops/cycles
consumes up to 192 Bytes/cycle

Core i7 Haswell:
Bandwidth
16 Bytes/cycle

Solution: Caches/Memory hierarchy

Typical Memory Hierarchy





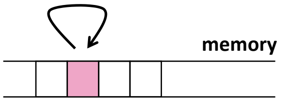
Why Caches Work: Locality

Locality: Programs tend to use data and instructions with addresses near or equal to those they have used recently

History of locality

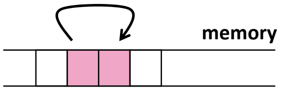
Temporal locality:

Recently referenced items are likely to be referenced again in the near future



Spatial locality:

Items with nearby addresses tend to be referenced close together in time



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Example: Locality?

```
sum = 0;
for (i = 0; i < n; i++)
  sum += a[i];
return sum;
```

Data:

- *Temporal:* ***sum*** *referenced in each iteration*
- *Spatial: array* ***a[ ]*** *accessed consecutively*

Instructions:

- *Temporal: loops cycle through the same instructions*
- *Spatial: instructions referenced in sequence*

Being able to assess the locality of code is a crucial skill for a performance programmer

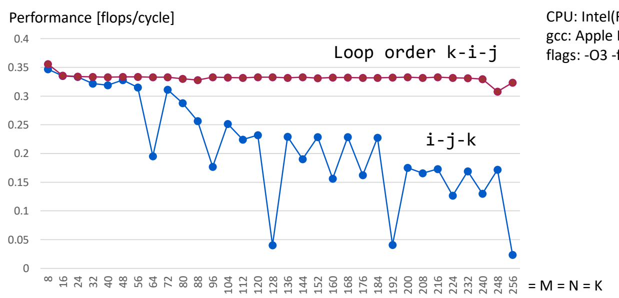
Locality Example #1

```
int sum_array_rows(double a[M][N])
{
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      sum += a[i][j];
  return sum;
}
```

Locality Example #2

```
int sum_array_3d(double a[K][M][N])
{
  int i, j, k, sum = 0;

  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      for (k = 0; k < K; k++)
        sum += a[k][i][j];
  return sum;
}
```

How to improve locality?



CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
gcc: Apple LLVM version 8.0.0 (clang-800.0.42.1)
flags: -O3 -fno-vectorize

Operational Intensity Again

Definition: Given a program P, assume cold (empty) cache

Operational intensity: $I(n) = \frac{W(n)}{Q(n)}$

- W(n): #flops (input size n)
- Q(n): #bytes transferred cache ↔ memory (for input size n)

Examples: Determine asymptotic bounds on I(n)

- *Vector sum: y = x + y* — **O(1)**
- *Matrix-vector product: y = Ax* — **O(1)**
- *Fast Fourier transform* — **O(log(n))**
- *Matrix-matrix product: C = AB + C* — **O(n)**

Compute/Memory Bound

A function/piece of code is:

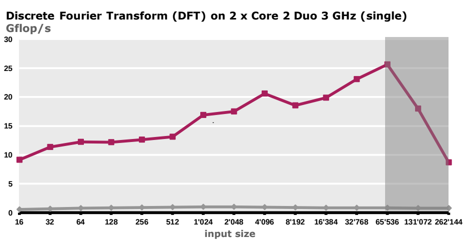
- ***Compute bound*** *if it has high operational intensity*
- ***Memory bound*** *if it has low operational intensity*

Relationship between operational intensity and locality?

- *They are closely related*
- *Operational intensity only describes the boundary last level cache/memory*

Effects

FFT: $I(n) = O(\log(n))$

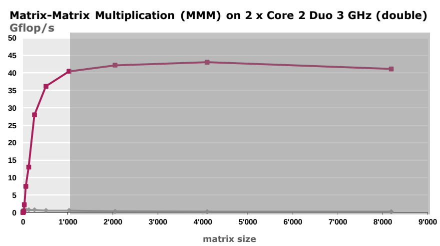


Up to 40-50% peak

Performance drop outside last level cache (LLC)

Most time spent transferring data

MMM: $I(n) = O(n)$



Up to 80-90% peak

Performance can be maintained outside LLC

Cache miss time compensated/hidden by computation

Cache

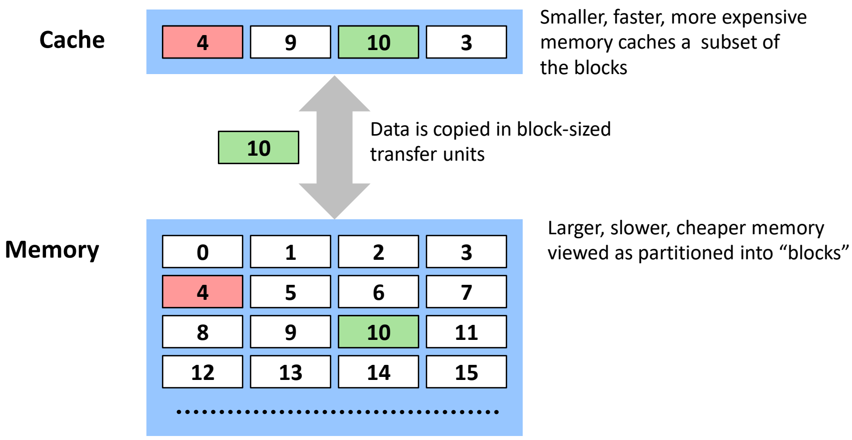
Definition: Computer memory with short access time used for the storage of frequently or recently used instructions or data

Naturally supports *temporal locality*

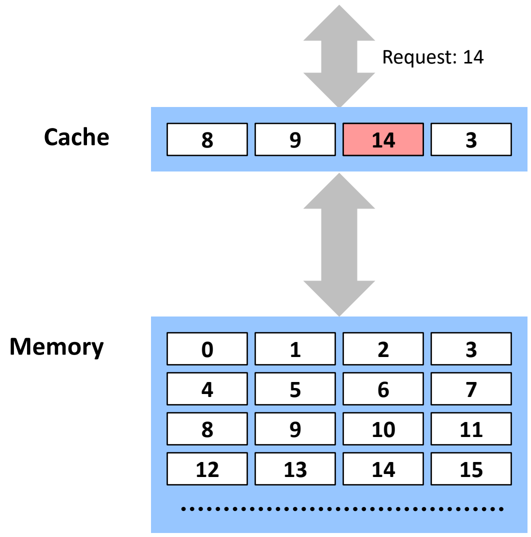
Spatial locality is supported by transferring data in blocks

- *Core family: one block = 64 B = 8 doubles*

General Cache Mechanics



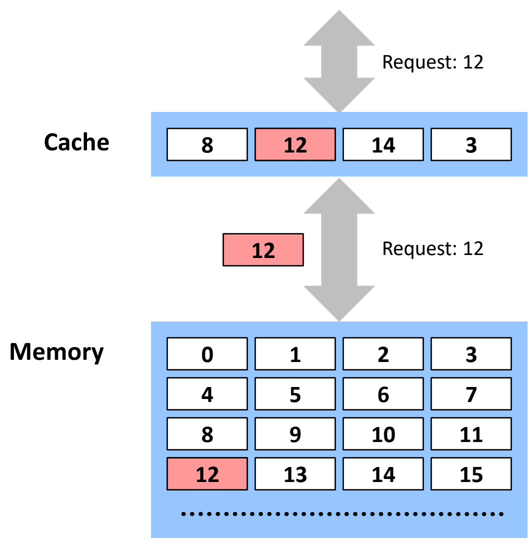
General Cache Concepts: Hit



Data in block b is needed

Block b is in cache:
Hit!

General Cache Concepts: Miss



Data in block b is needed

Block b is not in cache:
Miss!

Block b is fetched from memory

Block b is stored in cache
- *Placement policy:* determines where b goes
- *Replacement policy:* determines which block gets evicted (victim)

Types of Cache Misses (The 3 C’s)

Compulsory (cold) miss

Occurs on first access to a block

Capacity miss

Occurs when working set is larger than the cache

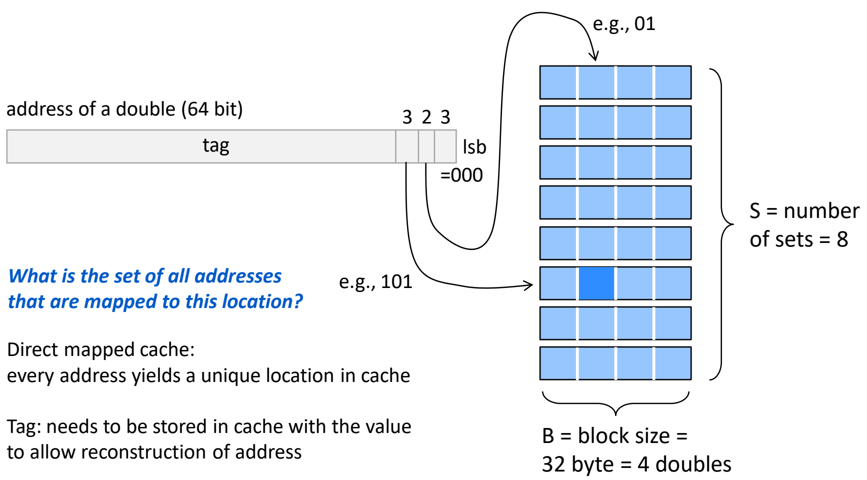
Conflict miss

Conflict misses occur when the cache is large enough, but multiple data objects all map to the same slot

Not a clean classification but still useful

Cache Structure

Example 1: direct mapped cache (E = 1, B = 4 doubles, S = 8)



What is the set of all addresses that are mapped to this location?

Direct mapped cache:
every address yields a unique location in cache

Tag: needs to be stored in cache with the value to allow reconstruction of address

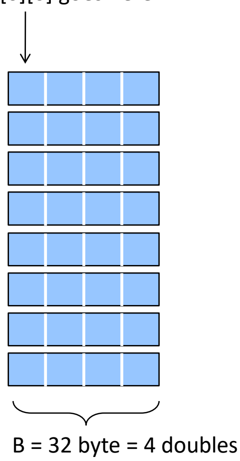
Always entire blocks (here 32 bytes) are loaded into cache

Example (S=8, E=1)

Ignore the variables sum, i, j

```
int sum_array_rows(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (i = 0; i < 16; i++)
    for (j = 0; j < 16; j++)
      sum += a[i][j];
  return sum;
}
```

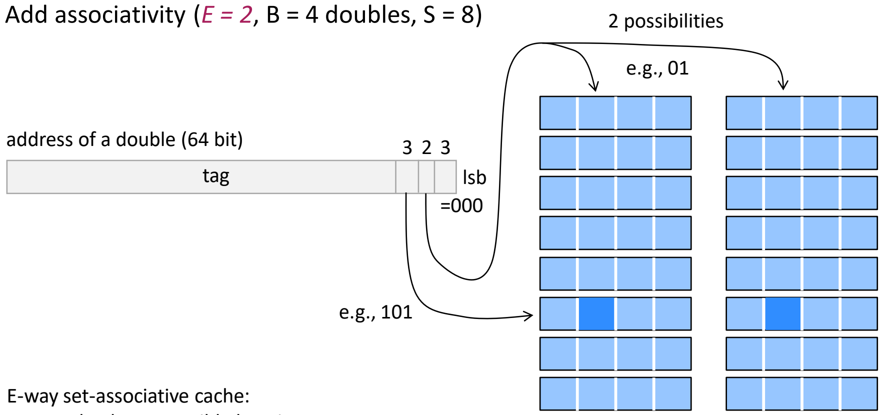
```
int sum_array_cols(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (j = 0; j < 16; j++)
    for (i = 0; i < 16; i++)
      sum += a[i][j];
  return sum;
}
```



How is the cache filled?

Cache Structure

Add associativity (*E = 2*, B = 4 doubles, S = 8)



E-way set-associative cache:
every value has E possible locations

Usually, least recently used (LRU) is replaced

Always entire blocks (here 32 bytes) are loaded into cache

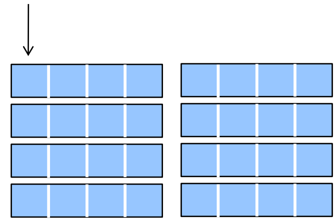
Example (S=4, E=2)

```
int sum_array_rows(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (i = 0; i < 16; i++)
    for (j = 0; j < 16; j++)
      sum += a[i][j];
  return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (j = 0; j < 16; j++)
    for (i = 0; i < 16; i++)
      sum += a[i][j];
  return sum;
}
```

Ignore the variables sum, i, j

assume: cold (empty) cache, a[0][0] goes here

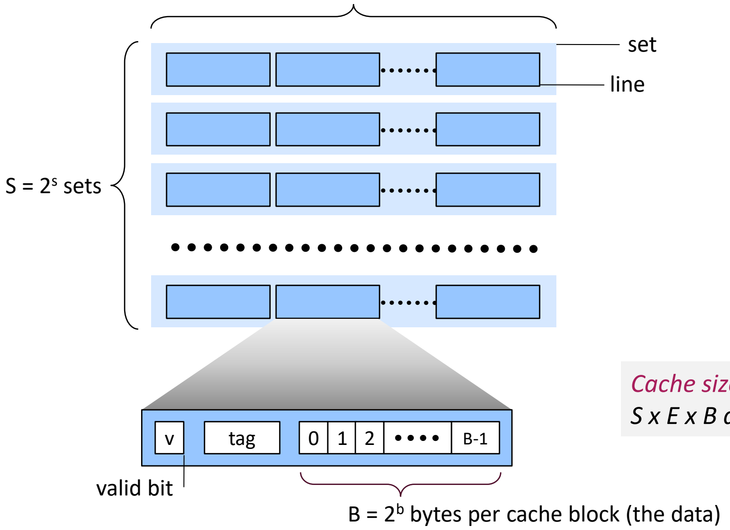


How is the cache filled?

General Cache Organization (S, E, B)

$E = 2^e$ lines per set

E = associativity, E=1: direct mapped

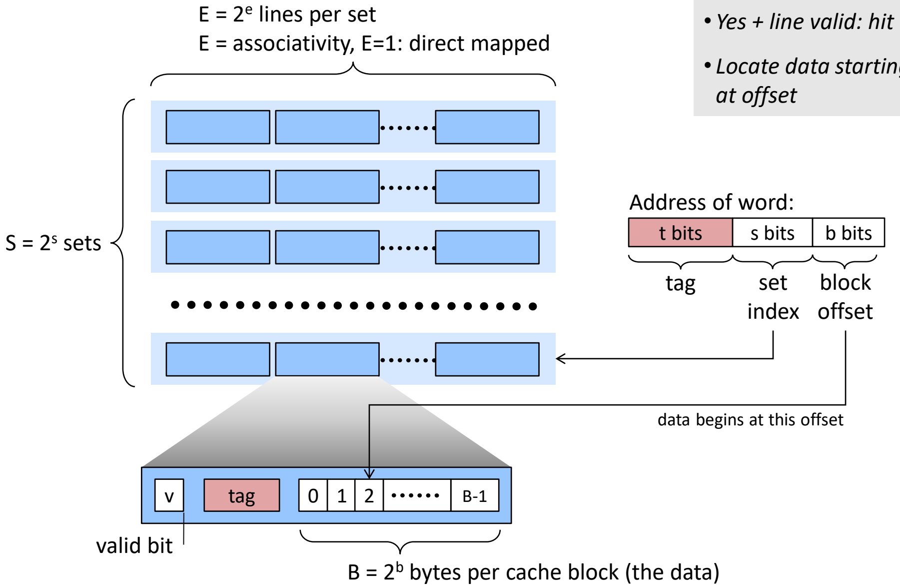


Cache size:

S x E x B data bytes

Cache Read

- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



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Terminology

Direct mapped cache:

- *Cache with E = 1*
- *Means every block from memory has a unique location in cache*

Fully associative cache

- *Cache with S = 1 (i.e., maximal E)*
- *Means every block from memory can be mapped to any location in cache*
- *In practice to expensive to build*
- *One can view the register file as a fully associative cache*

LRU (least recently used) replacement

- *when selecting which block should be replaced (happens only for E > 1), the least recently used one is chosen*

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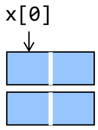
Small Example, Part 1



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for i = 0:7
    access(x[i])
```

Access pattern: 0123456701234567
Hit/Miss: MHMHMHMHMHMHMHMH

Result: 8 misses, 8 hits
Spatial locality: yes
Temporal locality: no

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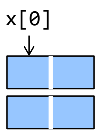
Small Example, Part 2



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for i = 0:2:7
    access(x[i])
  for i = 1:2:7
    access(x[i])
```

Access pattern: 0246135702461357
Hit/Miss: MMMMMMMMMMMMMMMM

Result: 16 misses
Spatial locality: no
Temporal locality: no

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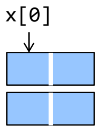
Small Example, Part 3



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for k = 0:1
    for i = 0:3
      access(x[i+4j])
```

Access pattern: 0123012345674567
Hit/Miss: MHMHHHHHMHMHHHHH

Result: 4 misses, 12 hits (is optimal, why?)
Spatial locality: yes
Temporal locality: yes

Cache Performance Metrics

Miss rate

- *Fraction of memory references not found in cache: misses / accesses*
  = 1 – hit rate

Hit time

- *Time to deliver a block in the cache to the processor*
- *Haswell:*
  4 clock cycles for L1
  11 clock cycles for L2

Miss penalty

- *Additional time required because of a miss*
- *Haswell: about 100 cycles for L3 miss*

What about writes?

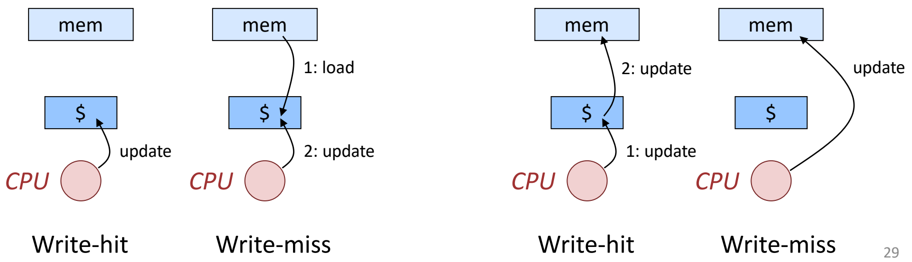
What to do on a write-hit?

- *Write-through: write immediately to memory*
- *Write-back: defer write to memory until replacement of line*

What to do on a write-miss?

- *Write-allocate: load into cache, update line in cache*
- *No-write-allocate: writes immediately to memory*

Write-back/write-allocate (Core)	*Write-through/no-write-allocate*



Example:

z = x + y, x, y, z vector of doubles of length n

assume they fit jointly in cache + cold cache

memory traffic Q(n): 4n doubles = 32n bytes

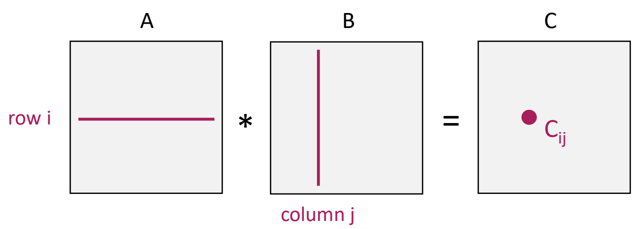
operational intensity I(n)? W(n) = n flops, so
I(n) = W(n)/Q(n) = 1/32

Locality Optimization: Blocking

Example: MMM

```
void mmm(double *A, double *B, double *C, int n) {

  for( int i = 0; i < n; i++ )
    for( int j = 0; j < n; j++ )
      for( int k = 0; k < n; k++ )
        C[n*i + j] = C[n*i + j] + A[n*i + k] * B[n*k + j]; }
```

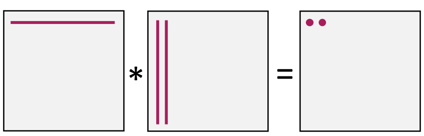


Cache Miss Analysis MMM

C = A*B, all n x n

Assumptions: cache size γ << n, cache block: 8 doubles, only 1 cache

Triple loop:

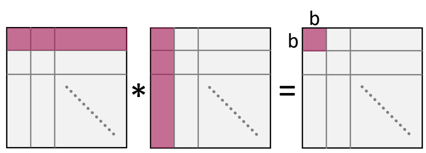


1. entry: n/8 + n = 9n/8 cache misses

2. entry: same

Total: $n^2 * 9n/8 = 9n^3/8$

Blocked (six-fold loop): block size b, 8 divides b



1. block: nb/8 + nb/8 = nb/4 cache misses

2. block: same

Total: $(n/b)^2 * nb/4 = n^3/(4b)$

How to choose b?

The above analysis assumes that the multiplication of b x b blocks can be done with only compulsory misses. This requires $3b^2 \le \gamma$.

b = sqrt(γ/3) which yields about sqrt(3)/(4*sqrt(γ)) * n^3 cache misses, a gain of ≈ 2.6*sqrt(γ)

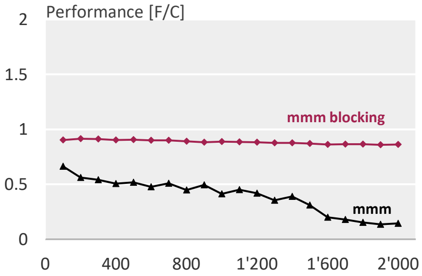
I(n) = O(sqrt(γ))

Experiment

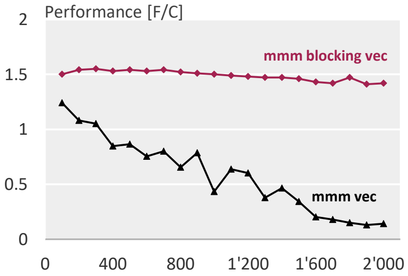
Cascade Lake (Intel® Xeon® Silver 4210)
GCC 9.3.0
Flags: -O3 -ffast-math [-fno-tree-vectorize] -march=native

L1 cache: 4096 doubles
Block size b = 32

Vectorization disabled



Vectorization enabled



On Previous Slide

Refine the analysis by including the misses incurred by C

Compute the operational intensity in both cases

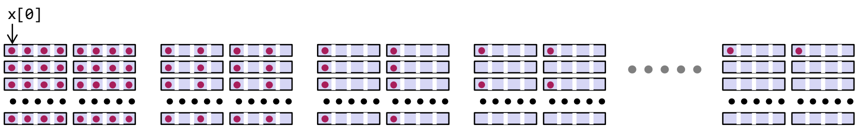
Try an analogous analysis for matrix-vector multiplication

The Killer: Two-Power Strided Working Sets

```
% t = 1,2,4,8,… a 2-power
% size W of working set: W = n/t
for (i = 0; i < n; i += t)
  access(x[i])
for (i = 0; i < n; i += t)
  access(x[i])
```

Cache: E = 2, B = 4 doubles

t = 1: t = 2: t = 4: t = 8: t ≥ 4S:



t = 1: Spatial locality
Temporal locality:
if W ≤ C

t = 2: Some spatial locality
Temporal locality:
if W ≤ C/2

t = 4: No spatial locality
Temporal locality:
if W ≤ C/4

t = 8: No spatial locality
Temporal locality:
if W ≤ C/8

t ≥ 4S: No spatial locality
Temporal locality:
if W ≤ 2

Working with a two-power-strided working set is like having a smaller cache

The Killer: Where Can It Occur?

Accessing two-power size 2D arrays (e.g., images) columnwise

- *2d Transforms*
- *Stencil computations*
- *Correlations*

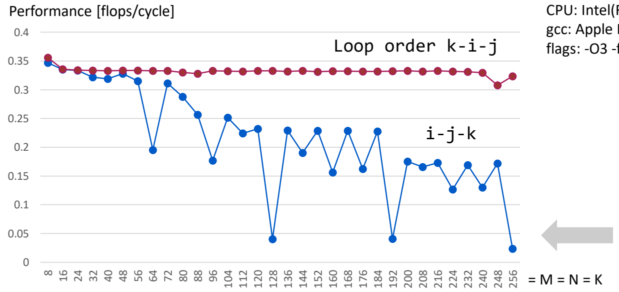
Various transform algorithms

- *Fast Fourier transform*
- *Wavelet transforms*
- *Filter banks*

Example from Before

```
int sum_array_3d(double a[K][M][N])
{
  int i, j, k, sum = 0;

  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      for (k = 0; k < K; k++)
        sum += a[k][i][j];
  return sum;
}
```



CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
gcc: Apple LLVM version 8.0.0 (clang-800.0.42.1)
flags: -O3 -fno-vectorize

2-power strides

Summary

It is important to assess temporal and spatial locality in the code

Cache structure is determined by three parameters

- *block size*
- *number of sets*
- *associativity*

You should be able to roughly simulate a computation on paper

Blocking to improve locality

Two-power strides can be problematic (conflict misses)