Advanced Systems Lab

Spring 2020

Lecture: DSL-based program generation for performance (Spiral)

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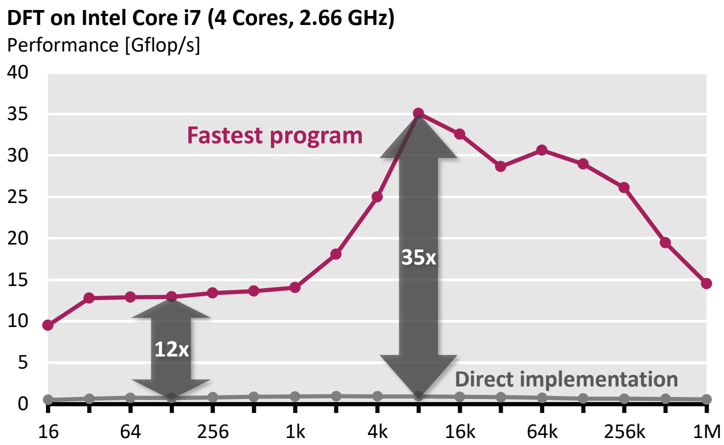
Spiral: DSL-Based Program Generation for Performance

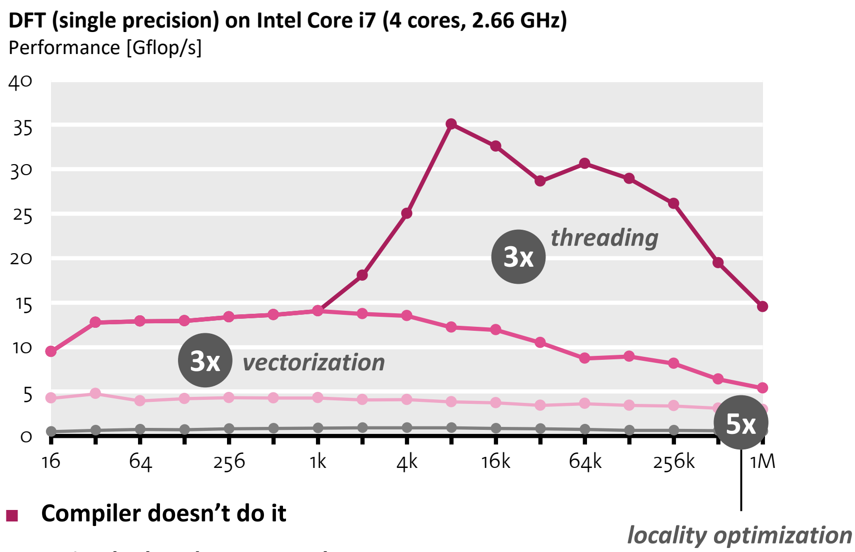
- www.spiral.net
- Franz Franchetti, Tze-Meng Low, Thom Popovici, Richard Veras, Daniele G. Spampinato, Jeremy Johnson, P, James C. Hoe and José M. F. Moura
  SPIRAL: Extreme Performance Portability
  Proceedings of the IEEE, special issue on ``From High Level Specification to High Performance Code'', Vol. 106, No. 11, 2018
- P, Franz Franchetti and Yevgen Voronenko
  Spiral
  in Encyclopedia of Parallel Computing, Eds. David Padua, pp. 1920-1933, Springer 2011
- P, José M. F. Moura, Jeremy Johnson, David Padua, Manuela Veloso, Bryan Singer, Jianxin Xiong, Franz Franchetti, Aca Gacic, Yevgen Voronenko, Kang Chen, Robert W. Johnson and Nicholas Rizzolo**,**
  SPIRAL: Code Generation for DSP Transforms
  Proceedings of the IEEE, special issue on «Program Generation, Optimization, and Adaptation'', Vol. 93, No. 2, pp. 232- 275, 2005

The Problem: Example DFT



- Same number of operations
- Best compiler

DFT: Analysis



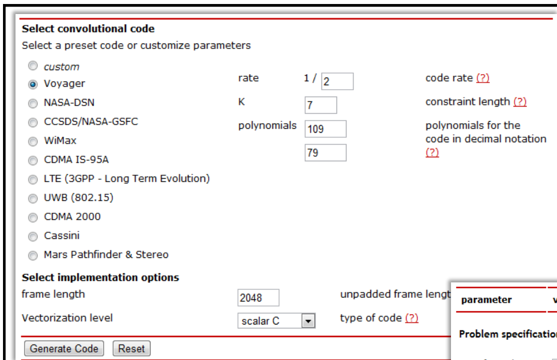
- Compiler doesn't do it
- Doing by hand: Very tough

Our Goal:

Computer writes high performance library code

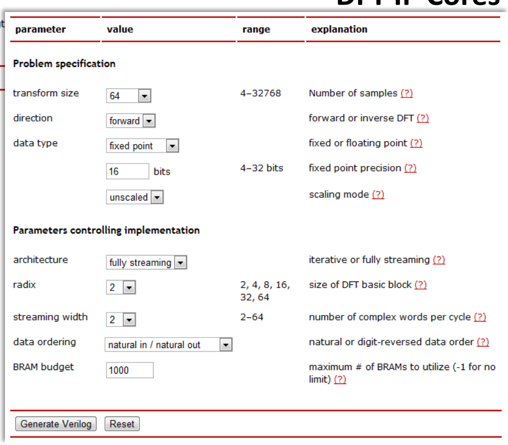
Generate Code

"click"



Viterbi Decoder

DFT IP Cores



@ www.spiral.net

Possible Approach:

Capturing algorithm knowledge:
Domain-specific languages (DSLs)

Structural optimization:
Rewriting systems

High performance code style:
Compiler

Decision making for choices:
Machine learning

Organization

- ***Spiral: Basic system***
- Vectorization
- General input size
- Results
- Final remarks

Algorithms: Example FFT, n = 4

Fast Fourier transform (FFT)

$$\begin{bmatrix}1&1&1&1\\1&i&-1&-i\\1&-1&1&-1\\1&-i&-1&i\end{bmatrix}x=\begin{bmatrix}1&\cdot&1&\cdot\\\cdot&1&\cdot&1\\1&\cdot&-1&\cdot\\\cdot&1&\cdot&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&\cdot&\cdot&i\end{bmatrix}\begin{bmatrix}1&1&\cdot&\cdot\\1&-1&\cdot&\cdot\\\cdot&\cdot&1&1\\\cdot&\cdot&1&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&\cdot&1\end{bmatrix}x$$

Representation using matrix algebra

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\,\mathrm{T}^4_2\,(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathrm{L}^4_2$$

- *SPL (Signal processing language):* **Mathematical, declarative, point-free**
- **Divide-and-conquer algorithms = breakdown rules in SPL**

Decomposition Rules (>200 for >40 Transforms)

$$\mathbf{DFT}_n \rightarrow P^\top_{k/2,2m}\left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\,\mathbf{rDFT}_{2m}(i/k)\right)\right)\left(\mathbf{RDFT}'_k \otimes I_m\right), \quad k \text{ even},$$

$$\begin{vmatrix}\mathbf{RDFT}_n\\\mathbf{RDFT}'_n\\\mathbf{DHT}_n\\\mathbf{DHT}'_n\end{vmatrix} \rightarrow (P^\top_{k/2,m} \otimes I_2)\left(\begin{vmatrix}\mathbf{RDFT}_{2m}\\\mathbf{RDFT}'_{2m}\\\mathbf{DHT}_{2m}\\\mathbf{DHT}'_{2m}\end{vmatrix} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\begin{vmatrix}\mathbf{rDFT}_{2m}(i/k)\\\mathbf{rDFT}_{2m}(i/k)\\\mathbf{rDHT}_{2m}(i/k)\\\mathbf{rDHT}_{2m}(i/k)\end{vmatrix}\right)\right)\left(\begin{vmatrix}\mathbf{RDFT}'_k\\\mathbf{RDFT}'_k\\\mathbf{DHT}'_k\\\mathbf{DHT}'_k\end{vmatrix} \otimes I_m\right), \quad k \text{ even},$$

$$\begin{vmatrix}\mathbf{rDFT}_{2n}(u)\\\mathbf{rDHT}_{2n}(u)\end{vmatrix} \rightarrow L^{2n}_m\left(I_k \otimes_i \begin{vmatrix}\mathbf{rDFT}_{2m}((i+u)/k)\\\mathbf{rDHT}_{2m}((i+u)/k)\end{vmatrix}\right)\left(\begin{vmatrix}\mathbf{rDFT}_{2k}(u)\\\mathbf{rDHT}_{2k}(u)\end{vmatrix} \otimes I_m\right),$$

$$\mathbf{RDFT\text{-}3}_n \rightarrow (Q^\top_{k/2,m} \otimes I_2)\,(I_k \otimes_i \mathbf{rDFT}_{2m})(i+1/2)/k))\,(\mathbf{RDFT\text{-}3}_k \otimes I_m), \quad k \text{ even},$$

$$\mathbf{DCT\text{-}2}_n \rightarrow P^\top_{k/2,2m}\left(\mathbf{DCT\text{-}2}_{2m}\,K^{2m}_2 \oplus \left(I_{k/2-1} \otimes N_{2m}\,\mathbf{RDFT\text{-}3}^\top_{2m}\right)\right)B_n(L^{n/2}_{k/2} \otimes I_2)(I_m \otimes \mathbf{RDFT}'_k)Q_{m/2,k},$$

$$\mathbf{DCT\text{-}3}_n \rightarrow \mathbf{DCT\text{-}2}^\top_n,$$

Decomposition rules = Algorithm knowledge in Spiral
(from ≈100 publications)

$$\mathbf{DFT}_n \rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, \quad n = km,\ \gcd(k,m) = 1$$

$$\mathbf{DCT\text{-}3}_n \rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\,\mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4))$$

$$\cdot(\mathrm{F}_2 \otimes \mathrm{I}_m)\begin{bmatrix}\mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1}\\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m)\end{bmatrix}, \quad n = 2m$$

$$\mathbf{DCT\text{-}4}_n \rightarrow S_n\mathbf{DCT\text{-}2}_n\,\mathrm{diag}_{0\le k<n}(1/(2\cos((2k+1)\pi/4n)))$$

$$\mathbf{IMDCT}_{2m} \rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix}1\\-1\end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix}-1\\-1\end{bmatrix} \otimes \mathrm{I}_m\right)\right)\mathrm{J}_{2m}\,\mathbf{DCT\text{-}4}_{2m}$$

$$\mathbf{WHT}_{2^k} \rightarrow \prod_{i=1}^{t}(\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t$$

$$\mathbf{DFT}_2 \rightarrow \mathrm{F}_2$$

$$\mathbf{DCT\text{-}2}_2 \rightarrow \mathrm{diag}(1, 1/\sqrt{2})\,\mathrm{F}_2$$

$$\mathbf{DCT\text{-}4}_2 \rightarrow \mathrm{J}_2\,\mathrm{R}_{13\pi/8}$$

Combining these rules yields many algorithms for every given transform

SPL to Code

SPL S	Pseudo code for $y = Sx$
$A_n B_n$	<code for: t = Bx> <code for: y = At>
$I_m \otimes A_n$	for (i=0; i<m; i++) <code for: y[i*n:1:i*n+n-1] = A(x[i*n:1:i*n+n-1])>
$A_m \otimes I_n$	for (i=0; i<n; i++) <code for: y[i:n:i+m*n-n] = A(x[i:n:i+m*n-n])>
D_n	for (i=0; i<n; i++) y[i] = D[i]*x[i];

L_k^{km}	for (i=0; i<k; i++)
	for (j=0; j<m; j++)
	y[i*m+j] = x[j*k+i];

F_2	y[0] = x[0] + x[1]; y[1] = x[0] - x[1];

$$\mathrm{I}_m \otimes A_n = \begin{bmatrix} A_n & & \\ & \ddots & \\ & & A_n \end{bmatrix}$$

Correct code: ***easy*** *fast code:* ***very difficult***

Program Generation in Spiral

Transform $\mathbf{DFT}_8$

Decomposition rules

Algorithm *(SPL)* $(\mathbf{DFT}_2 \otimes I_4)\, T_4^8\, (I_2 \otimes ((\mathbf{DFT}_2 \otimes I_2)\, T_2^4\, (I_2 \otimes \mathbf{DFT}_2)\, L_2^4))\, L_2^8$

parallelization
vectorization

Algorithm *(∑-SPL)* $\sum\left(S_j\, \mathbf{DFT}_2\, G_j\right) \sum\left(\sum\left(S_{k,l}\, \mathrm{diag}(t_{k,l})\, \mathbf{DFT}_2\, G_l\right) \sum\left(S_m\, \mathrm{diag}(t_m)\, \mathbf{DFT}_2\, G_{k,m}\right)\right)$

locality
optimization

C Program

```
void sub(double *y, double *x) {
double f0, f1, f2, f3, f4, f7, f8, f10, f11;
    f0 = x[0] - x[3];
    f1 = x[0] + x[3];
    f2 = x[1] - x[2];
    f3 = x[1] + x[2];
    f4 = f1 - f3;
    y[0] = f1 + f3;
    y[2] = 0.7071067811865476 * f4;
    f7 = 0.9238795325112867 * f0;
< more lines>
```

basic block
optimizations

+ search or learning
for further tuning

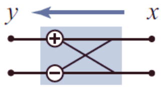
Organization

- Spiral: Basic system
- ***Vectorization***
- General input size
- Results
- Final remarks

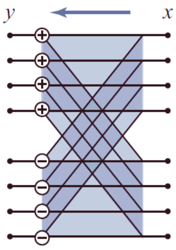
Example: Vectorization in Spiral

- **Goal: Translate SPL expressions directly into SIMD code**
- **Relationship SPL expressions ↔ vectorization?**

$$y = \mathbf{DFT}_2 \, x$$



$$y = \left(\mathbf{DFT}_2 \otimes \mathrm{I}_4\right) x$$



one addition
one subtraction

one (4-way) vector addition
one (4-way) vector subtraction

Step 1: Identify "Good" Vector Constructs

- **Vector length:** ν
- **Good (= easily vectorizable) SPL constructs:**

  $A \otimes \mathrm{I}_\nu$

  $\mathrm{L}_\nu^{\nu^2}, \ \mathrm{L}_2^{2\nu}, \mathrm{L}_\nu^{2\nu}$ *base cases*

  SPL expressions recursively built from those

- ***Idea:*** **Convert a given SPL expression into a "good" SPL expression through rewriting (structural manipulation)**

Step 2: Find Manipulation Rules

$$
\begin{array}{rcl}
\mathrm{L}_n^{n\nu} & \to & \left(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_\nu^{\nu^2}\right)\left(\mathrm{L}_{n/\nu}^{n} \otimes \mathrm{I}_\nu\right) \\
\mathrm{L}_\nu^{n\nu} & \to & \left(\mathrm{L}_\nu^{n} \otimes \mathrm{I}_\nu\right)\left(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_\nu^{\nu^2}\right) \\
\mathrm{L}_m^{mn} & \to & \left(\mathrm{L}_m^{mn/\nu} \otimes \mathrm{I}_\nu\right)\left(\mathrm{I}_{mn/\nu^2} \otimes \mathrm{L}_\nu^{\nu^2}\right)\left(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_{m/\nu}^{m} \otimes \mathrm{I}_\nu\right) \\
\mathrm{I}_l \otimes \mathrm{L}_n^{kmn} \otimes \mathrm{I}_r & \to & \left(\mathrm{I}_l \otimes \mathrm{L}_n^{kn} \otimes \mathrm{I}_{mr}\right)\left(\mathrm{I}_{kl} \otimes \mathrm{L}_n^{mn} \otimes \mathrm{I}_r\right) \\
\mathrm{I}_l \otimes \mathrm{L}_n^{kmn} \otimes \mathrm{I}_r & \to & \left(\mathrm{I}_l \otimes \mathrm{L}_{kn}^{kmn} \otimes \mathrm{I}_r\right)\left(\mathrm{I}_l \otimes \mathrm{L}_{mn}^{kmn} \otimes \mathrm{I}_r\right) \\
\mathrm{I}_l \otimes \mathrm{L}_{km}^{kmn} \otimes \mathrm{I}_r & \to & \left(\mathrm{I}_{kl} \otimes \mathrm{L}_m^{mn} \otimes \mathrm{I}_r\right)\left(\mathrm{I}_l \otimes \mathrm{L}_k^{kn} \otimes \mathrm{I}_{mr}\right) \\
\mathrm{I}_l \otimes \mathrm{L}_{km}^{kmn} \otimes \mathrm{I}_r & \to & \left(\mathrm{I}_l \otimes \mathrm{L}_k^{kmn} \otimes \mathrm{I}_r\right)\left(\mathrm{I}_l \otimes \mathrm{L}_m^{kmn} \otimes \mathrm{I}_r\right)
\end{array}
$$

Manipulation rules = SIMD knowledge in Spiral

$$
\begin{array}{rcl}
\left(\mathrm{I}_m \otimes A^{n\times n}\right)\mathrm{L}_m^{mn} & \to & \left(\mathrm{I}_{m/\nu} \otimes \mathrm{L}_\nu^{n\nu}\left(A^{n\times n} \otimes \mathrm{I}_\nu\right)\right)\left(\mathrm{L}_{m/\nu}^{mn/\nu} \otimes \mathrm{I}_\nu\right) \\
\mathrm{L}_n^{mn}\left(\mathrm{I}_m \otimes A^{n\times n}\right) & \to & \left(\mathrm{L}_n^{mn/\nu} \otimes \mathrm{I}_\nu\right)\left(\mathrm{I}_{m/\nu} \otimes \left(A^{n\times n} \otimes \mathrm{I}_\nu\right)\mathrm{L}_n^{n\nu}\right) \\
\left(\mathrm{I}_k \otimes \left(\mathrm{I}_m \otimes A^{n\times n}\right)\mathrm{L}_m^{mn}\right)\mathrm{L}_k^{kmn} & \to & \left(\mathrm{L}_k^{km} \otimes \mathrm{I}_n\right)\left(\mathrm{I}_m \otimes \left(\mathrm{I}_k \otimes A^{n\times n}\right)\mathrm{L}_k^{kn}\right)\left(\mathrm{L}_m^{mn} \otimes \mathrm{I}_k\right) \\
\mathrm{L}_{mn}^{kmn}\left(\mathrm{I}_k \otimes \mathrm{L}_n^{mn}\left(\mathrm{I}_m \otimes A^{n\times n}\right)\right) & \to & \left(\mathrm{L}_n^{mn} \otimes \mathrm{I}_k\right)\left(\mathrm{I}_m \otimes \mathrm{L}_n^{kn}\left(\mathrm{I}_k \otimes A^{n\times n}\right)\right)\left(\mathrm{L}_m^{km} \otimes \mathrm{I}_n\right) \\
\overline{A\,B} & \to & \overline{A}\,\overline{B} \\
\overline{A^{m\times m} \otimes \mathrm{I}_\nu} & \to & \left(\mathrm{I}_m \otimes \mathrm{L}_\nu^{2\nu}\right)\left(\overline{A^{m\times m}} \otimes \mathrm{I}_\nu\right)\left(\mathrm{I}_m \otimes \mathrm{L}_2^{2\nu}\right) \\
\overline{\mathrm{I}_m \otimes A^{n\times n}} & \to & \mathrm{I}_m \otimes \overline{A^{n\times n}} \\
\overline{D} & \to & \left(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_\nu^{2\nu}\right)\vec{D}\left(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_2^{2\nu}\right) \\
\overline{P} & \to & P \otimes \mathrm{I}_2
\end{array}
$$

Example

$$\underbrace{\mathbf{DFT}_{mn}}_{\mathrm{vec}(\nu)} \rightarrow \underbrace{(\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathsf{T}_n^{mn}(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathsf{L}_m^{mn}}_{\mathrm{vec}(\nu)}$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

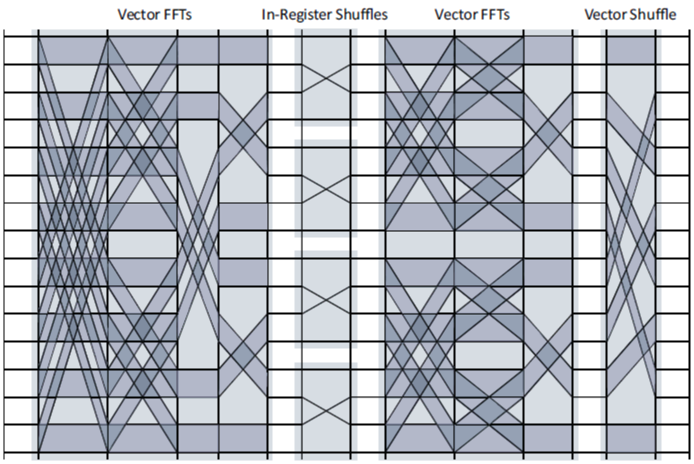
$$\rightarrow \underline{\left(\mathrm{I}_{\frac{mn}{\nu}} \otimes \mathsf{L}_\nu^{2\nu}\right)}\ \underline{\left(\overline{\mathbf{DFT}_m \otimes \mathrm{I}_{\frac{n}{\nu}}} \otimes \mathrm{I}_\nu\right)}\ \overline{\mathsf{T}}{}'^{mn}_{n}$$

$$\underline{\left(\mathrm{I}_{\frac{m}{\nu}} \otimes \left(\mathsf{L}_\nu^{2n} \otimes \mathrm{I}_\nu\right)\left(\mathrm{I}_{\frac{2n}{\nu}} \otimes \mathsf{L}_\nu^{\nu^2}\right)\left(\mathrm{I}_{\frac{n}{\nu}} \otimes \mathsf{L}_2^{2\nu} \otimes \mathrm{I}_\nu\right)\right.}\ \underline{\left.\left(\overline{\mathbf{DFT}}_n \otimes \mathrm{I}_\nu\right)\right)}\ \underline{\left(\mathsf{L}_{\frac{m}{\nu}}^{\frac{mn}{\nu}} \otimes \mathsf{L}_2^{2\nu}\right)}$$

vectorized arithmetic

vectorized data accesses

Sketch for complex ν = 2



$$\Big(((\mathrm{DFT}_2 \otimes I_2)T_2^4(I_2 \otimes \mathrm{DFT}_2)L_2^4) \otimes I_2) \otimes I_2\Big)T_4^{16}\Big(I_2 \otimes (L_2^4 \otimes I_2)(I_2 \otimes L_2^4)((\mathrm{DFT}_2 \otimes I_2)T_2^4(I_2 \otimes \mathrm{DFT}_2)L_2^4) \otimes I_2\Big)(L_2^8 \otimes I_2)$$

Automatically Generate Base Case Library

- *Goal:* **Given instruction set, generate base cases**

  $\nu = 4: \quad \{\mathrm{L}_2^4,\ \mathrm{I}_2 \otimes \mathrm{L}_2^4,\ \mathrm{L}_2^4 \otimes \mathrm{I}_2,\ \mathrm{L}_2^8,\ \mathrm{L}_4^8\}$

- *Idea:* **Instructions as matrices + search**

```
y = _mm_unpacklo_ps(x0, x1);
```

$$y = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix} \begin{bmatrix} \vec{x}_0 \\ \vec{x}_1 \end{bmatrix}$$

```
y = _mm_shuffle_ps(x0, x1, _MM_SHUFFLE(1,2,1,2));
```

$$y = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix} \begin{bmatrix} \vec{x}_0 \\ \vec{x}_1 \end{bmatrix}$$

```
y = _mm_shuffle_ps(x0, x1, _MM_SHUFFLE(3,4,3,4));
```

$$y = \begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} \vec{x}_0 \\ \vec{x}_1 \end{bmatrix}$$

$$\left[\begin{array}{cccccccc} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ \hline 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{array}\right]$$

```
y0 = _mm_shuffle_ps(x0, x1,
         _MM_SHUFFLE(1,2,1,2));   );
y1 = _mm_shuffle_ps(x0, x1,
         _MM_SHUFFLE(3,4,3,4));   ;
```

$\mathrm{L}_2^4 \otimes \mathrm{I}_2$

No base case

Base case

Same Approach for Different Paradigms

Threading:

$$\underbrace{\mathbf{DFT}_{mn}}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{\left((\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathrm{T}_n^{mn}(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathrm{L}_m^{mn}\right)}_{\mathsf{smp}(p,\mu)}$$

$$\cdots$$

$$\rightarrow \underbrace{\left(\mathbf{DFT}_m \otimes \mathrm{I}_n\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\mathrm{T}_n^{mn}}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_m \otimes \mathbf{DFT}_n\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\mathrm{L}_m^{nm}}_{\mathsf{smp}(p,\mu)}$$

$$\cdots$$

$$\rightarrow \left((\mathrm{L}_m^{mp} \otimes \mathrm{I}_{n/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)\left(\mathrm{I}_p \otimes_\| (\mathbf{DFT}_m \otimes \mathrm{I}_{n/p})\right)\left((\mathrm{L}_p^{mp} \otimes \mathrm{I}_{n/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)$$
$$\left(\bigoplus_{i=0}^{p-1}{}_\| \, \mathrm{T}_n^{mn,i}\right)\left(\mathrm{I}_p \otimes_\| (\mathrm{I}_{m/p} \otimes \mathbf{DFT}_n)\right)\left(\mathrm{I}_p \otimes_\| \mathrm{L}_{m/p}^{mn/p}\right)\left((\mathrm{L}_p^{pm} \otimes \mathrm{I}_{m/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)$$

Vectorization:

$$\underbrace{\left(\overline{\mathbf{DFT}_{mn}}\right)}_{\mathsf{vec}(\nu)} \rightarrow \underbrace{\left((\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathrm{T}_n^{mn}(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathrm{L}_m^{mn}\right)}_{\mathsf{vec}(\nu)}$$

$$\cdots$$

$$\rightarrow \underbrace{\overleftarrow{(\mathrm{DFT}_m \otimes \mathrm{I}_n)}^{\nu}}_{\mathsf{vec}(\nu)} \underbrace{(\mathrm{T}_n^{mn})^{\nu}}_{\mathsf{vec}(\nu)} \underbrace{\overrightarrow{(\mathrm{I}_m \otimes \mathrm{DFT}_n)\,\mathrm{L}_m^{mn}}^{\nu}}_{\mathsf{vec}(\nu)}$$

$$\cdots$$

$$\rightarrow (\mathrm{I}_{mn/\nu} \otimes \underbrace{\mathrm{L}_\nu^{2\nu}}_{\mathsf{sse}})(\overline{\mathrm{DFT}_m \otimes \mathrm{I}_{n/\nu}} \vec{\otimes} \mathrm{I}_\nu)\underbrace{\overline{(\mathrm{T}_n^{mn})}^{\nu}}_{\mathsf{sse}}$$
$$\left(\mathrm{I}_{m/\nu} \otimes (\overline{\mathrm{L}_\nu^n} \vec{\otimes} \mathrm{I}_\nu)(\mathrm{I}_{n/\nu} \otimes (\mathrm{L}_\nu^{2\nu} \vec{\otimes} \mathrm{I}_\nu)(\mathrm{I}_2 \otimes \underbrace{\mathrm{L}_\nu^{\nu^2}}_{\mathsf{sse}})(\mathrm{L}_2^{2\nu} \vec{\otimes} \mathrm{I}_\nu))(\overline{\mathrm{DFT}_n} \vec{\otimes} \mathrm{I}_\nu)\right)$$
$$\left((\mathrm{L}_m^{mn} \otimes \mathrm{I}_2) \vec{\otimes} \mathrm{I}_\nu\right)(\mathrm{I}_{mn/\nu} \otimes \underbrace{\mathrm{L}_2^{2\nu}}_{\mathsf{sse}})$$

GPUs:

$$\underbrace{\left(\mathbf{DFT}_{r^k}\right)}_{\mathsf{gpu}(t,c)} \rightarrow \underbrace{\left(\prod_{i=0}^{k-1} \mathrm{L}_r^{r^k}\left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)\left(\mathrm{L}_{r^{k-i-1}}^{r^k}(\mathrm{I}_{r^i} \otimes \mathrm{T}_{r^{k-i-1}}^{r^{k-i}})\underbrace{\mathrm{L}_{r^{i+1}}^{r^k}}_{\mathsf{vec}(c)}\right)\right)\mathrm{R}_r^{r^k}}_{\mathsf{gpu}(t,c)}$$

$$\cdots$$

$$\rightarrow \left(\prod_{i=0}^{k-1} (\mathrm{L}_r^{r^n/2} \vec{\otimes} \mathrm{I}_2)\left(\mathrm{I}_{r^{n-1}/2} \otimes_\times \underbrace{(\mathbf{DFT}_r \vec{\otimes} \mathrm{I}_2)\,\mathrm{L}_r^{2r}}_{\mathsf{shd}(t,c)}\right)\mathrm{T}_i\right)$$
$$(\mathrm{L}_r^{r^n/2} \vec{\otimes} \mathrm{I}_2)(\mathrm{I}_{r^{n-1}/2} \otimes_\times \underbrace{\mathrm{L}_r^{2r}}_{\mathsf{shd}(t,c)})(\mathrm{R}_r^{r^{n-1}} \vec{\otimes} \mathrm{I}_r)$$

Verilog for FPGAs:

$$\underbrace{\left(\mathbf{DFT}_{r^k}\right)}_{\mathsf{stream}(r^s)} \rightarrow \underbrace{\left[\prod_{i=0}^{k-1} \mathrm{L}_r^{r^k}\left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)\left(\mathrm{L}_{r^{k-i-1}}^{r^k}(\mathrm{I}_{r^i} \otimes \mathrm{T}_{r^{k-i-1}}^{r^{k-i}})\,\mathrm{L}_{r^{i+1}}^{r^k}\right)\right]\mathrm{R}_r^{r^k}}_{\mathsf{stream}(r^s)}$$

$$\cdots$$

$$\rightarrow \left[\prod_{i=0}^{k-1} \underbrace{\mathrm{L}_r^{r^k}}_{\mathsf{stream}(r^s)} \underbrace{\left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)}_{\mathsf{stream}(r^s)} \underbrace{\left(\mathrm{L}_{r^{k-i-1}}^{r^k}(\mathrm{I}_{r^i} \otimes \mathrm{T}_{r^{k-i-1}}^{r^{k-i}})\,\mathrm{L}_{r^{i+1}}^{r^k}\right)}_{\mathsf{stream}(r^s)}\right] \underbrace{\mathrm{R}_r^{r^k}}_{\mathsf{stream}(r^s)}$$

$$\cdots$$

$$\rightarrow \left[\prod_{i=0}^{k-1} \underbrace{\mathrm{L}_r^{r^k}}_{\mathsf{stream}(r^s)} \left(\mathrm{I}_{r^{k-s-1}} \otimes_s (\mathrm{I}_{r^{s-1}} \otimes \mathbf{DFT}_r)\right) \underbrace{\mathrm{T}_i'}_{\mathsf{stream}(r^s)}\right] \underbrace{\mathrm{R}_r^{r^k}}_{\mathsf{stream}(r^s)}$$

- Rigorous, correct by construction
- Overcomes compiler limitations

Organization

- Spiral: Basic system
- Vectorization
- ***General input size***
- Results
- Final remarks

Challenge: General Size Libraries

So far:
Code specialized to fixed input size

```
DFT_384(x, y) {
  …
  for(i = …) {
   t[2i]   = x[2i] + x[2i+1]
   t[2i+1] = x[2i] - x[2i+1]
  }
  …
}
```

- Algorithm fixed
- Nonrecursive code

Challenge:
Library for general input size

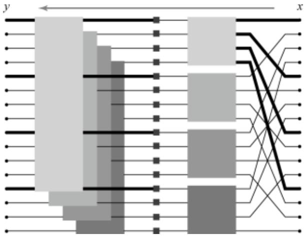
```
DFT(n, x, y) {
  …
  for(i = …) {
    DFT_strided(m, x+mi, y+i, 1, k)
  }
  …
}
```

- Algorithm cannot be fixed
- Recursive code
- Creates many challenges

Challenge: Recursion Steps

- **Cooley-Tukey FFT**

$$y = (\mathbf{DFT}_k \otimes I_m) T_m^{km} (I_k \otimes \mathbf{DFT}_m) L_k^{km} x$$



- **Implementation that increases locality (e.g., FFTW 2.x)**

```
void DFT(int n, cpx *y, cpx *x) {
  int k = choose_dft_radix(n);
  …
  for (int i=0; i < k; ++i)
    DFTrec(m, y + m*i, x + i, k, 1);
  for (int j=0; j < m; ++j)
    DFTscaled(k, y + j, t[j], m);
}
```

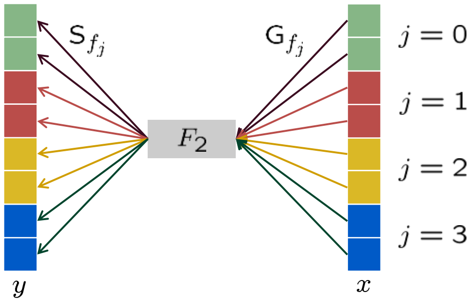
Σ–SPL : Basic Idea

- **Four additional matrix constructs: Σ, G, S, Perm**
  - **Σ (sum)** — **explicit loop**
  - **G_f (gather)** — **load data with index mapping f**
  - **S_f (scatter)** — **store data with index mapping f**
  - **Perm_f** — **permute data with the index mapping f**

- **Σ-SPL formulas = matrix factorizations**

Example: $y = (I_4 \otimes F_2)x \;\rightarrow\; y = \sum_{j=0}^{3} \mathsf{S}_{f_j} F_2 \mathsf{G}_{f_j} x$

$$y = \begin{bmatrix} F_2 & & & \\ & F_2 & & \\ & & F_2 & \\ & & & F_2 \end{bmatrix} x$$



Find Recursion Step Closure

Voronenko, 2008

$$\{\mathbf{DFT}_n\}$$

$$(\{\mathbf{DFT}_{n/k}\} \otimes I_k) T_k^n (I_{n/k} \otimes \{\mathbf{DFT}_k\}) L_{n/k}^n$$

$$\left(\sum_{i=0}^{k-1} \mathsf{S}_{h_{i,k}} \{\mathbf{DFT}_{n/k}\}\, \mathsf{G}_{h_{i,k}}\right) \mathsf{diag}(f) \left(\sum_{j=0}^{n/k-1} \mathsf{S}_{h_{jk,1}} \{\mathbf{DFT}_k\}\, \mathsf{G}_{h_{jk,1}}\right) \mathsf{perm}(\ell_{n/k}^n)$$

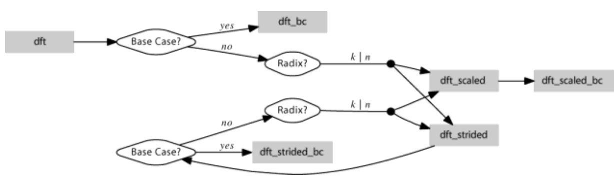
$$\sum_{i=0}^{k-1} \mathsf{S}_{h_{i,k}} \{\mathbf{DFT}_{n/k}\}\, \mathsf{diag}(f \circ h_{i,k})\, \mathsf{G}_{h_{i,k}} \sum_{j=0}^{n/k-1} \mathsf{S}_{h_{jk,1}} \{\mathbf{DFT}_k\}\, \mathsf{G}_{h_{j,n/k}}$$

$$\sum_{i=0}^{k-1} \left\{ \mathsf{S}_{h_{i,k}}\, \mathbf{DFT}_{n/k}\, \mathsf{diag}(f \circ h_{i,k})\, \mathsf{G}_{h_{i,k}} \right\} \sum_{j=0}^{n/k-1} \left\{ \mathsf{S}_{h_{jk,1}}\, \mathbf{DFT}_k\, \mathsf{G}_{h_{j,n/k}} \right\}$$

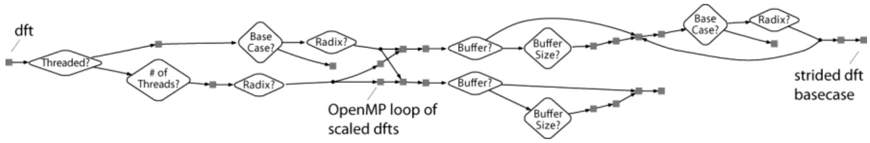
Repeat until closure

Recursion Step Closure: Examples

DFT: scalar code (like FFTW 2.x)



DFT: full-fledged (vectorized and parallel code)



Summary: Complete Automation for Transforms

- **Memory hierarchy optimization**
  Rewriting and search for algorithm selection
  Rewriting for loop optimizations
- **Vectorization**
  Rewriting
- **Parallelization**
  Rewriting

fixed input size code

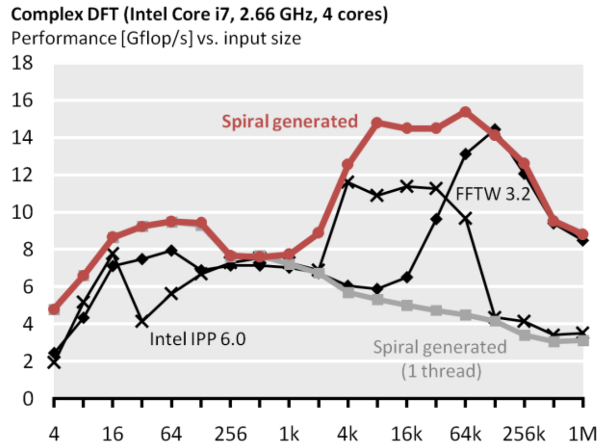
- **Derivation of library structure**
  Rewriting
  Other methods

general input size library

Organization

- Spiral: Basic system
- Vectorization
- General input size
- ***Results***
- Final remarks

DFT on Intel Multicore



$$\mathbf{DFT}_n \to (\mathbf{DFT}_k \otimes I_m) T_m^n (I_k \otimes \mathbf{DFT}_m) L_k^n$$
$$\mathbf{DFT}_n \to P_{k/2,2m}^\top \left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{RDFT}_n \to (P_{k/2,m}^\top \otimes I_2) \left(\mathbf{RDFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{rDFT}_{2n}(u) \to L_m^{2n} \left(I_k \otimes_i \mathbf{rDFT}_{2m}((i+u)/k)\right) (\mathbf{rDFT}_{2k}(u) \otimes I_m)$$

Spiral

5MB vectorized, threaded, general-size, adaptive library

Generating 100s of FFTWs

PhD thesis Voronenko, 2009

$$\mathbf{DFT}_n \to P_{k/2,2m}^\top \left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) \left(\mathbf{RDFT}'_k \otimes I_m\right), \quad k \text{ even},$$

$$\left|\begin{matrix}\mathbf{RDFT}_n \\ \mathbf{RDFT}'_n \\ \mathbf{DHT}_n \\ \mathbf{DHT}'_n\end{matrix}\right| \to (P_{k/2,m}^\top \otimes I_2) \left(\left|\begin{matrix}\mathbf{RDFT}_{2m} \\ \mathbf{RDFT}'_{2m} \\ \mathbf{DHT}_{2m} \\ \mathbf{DHT}'_{2m}\end{matrix}\right| \oplus \left(I_{k/2-1} \otimes_i D_{2m} \left|\begin{matrix}\mathbf{rDFT}_{2m}(i/k) \\ \mathbf{rDFT}_{2m}(i/k) \\ \mathbf{rDHT}_{2m}(i/k) \\ \mathbf{rDHT}_{2m}(i/k)\end{matrix}\right|\right)\right) \left(\left|\begin{matrix}\mathbf{RDFT}'_k \\ \mathbf{RDFT}'_k \\ \mathbf{DHT}'_k \\ \mathbf{DHT}'_k\end{matrix}\right| \otimes I_m\right), \quad k \text{ even},$$

$$\left|\begin{matrix}\mathbf{rDFT}_{2n}(u) \\ \mathbf{rDHT}_{2n}(u)\end{matrix}\right| \to L_m^{2n} \left(I_k \otimes_i \left|\begin{matrix}\mathbf{rDFT}_{2m}((i+u)/k) \\ \mathbf{rDHT}_{2m}((i+u)/k)\end{matrix}\right|\right) \left(\left|\begin{matrix}\mathbf{rDFT}_{2k}(u) \\ \mathbf{rDHT}_{2k}(u)\end{matrix}\right| \otimes I_m\right),$$

$$\mathbf{RDFT\text{-}3}_n \to (Q_{k/2,m}^\top \otimes I_2)\, (I_k \otimes_i \mathbf{rDFT}_{2m})(i+1/2)/k))\, (\mathbf{RDFT\text{-}3}_k \otimes I_m), \quad k \text{ even},$$

$$\mathbf{DCT\text{-}2}_n \to P_{k/2,2m}^\top \left(\mathbf{DCT\text{-}2}_{2m} K_2^{2m} \oplus \left(I_{k/2-1} \otimes N_{2m}\, \mathbf{RDFT\text{-}3}_{2m}^\top\right)\right) B_n (L_{k/2}^{n/2} \otimes I_2)(I_m \otimes \mathbf{RDFT}'_k) Q_{m/2,k},$$

$$\mathbf{DCT\text{-}3}_n \to \mathbf{DCT\text{-}2}_n^\top,$$

$$\mathbf{DCT\text{-}4}_n \to Q_{k/2,2m}^\top \left(I_{k/2} \otimes N_{2m}\, \mathbf{RDFT\text{-}3}_{2m}^\top\right) B'_n (L_{k/2}^{n/2} \otimes I_2)(I_m \otimes \mathbf{RDFT\text{-}3}_k) Q_{m/2,k}.$$

$$\mathbf{DFT}_n \to (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathsf{T}_m^n (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathsf{L}_k^n, \quad n = km$$
$$\mathbf{DFT}_n \to P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1$$
$$\mathbf{DFT}_p \to R_p^T (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime}$$
$$\mathbf{DCT\text{-}3}_n \to (\mathrm{I}_m \oplus \mathsf{J}_m)\, \mathsf{L}_m^n (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \cdot (\mathsf{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathsf{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m$$
$$\mathbf{DCT\text{-}4}_n \to S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n)))$$
$$\mathbf{IMDCT}_{2m} \to (\mathsf{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathsf{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right) \mathsf{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m}$$
$$\mathbf{WHT}_{2^k} \to \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t$$
$$\mathbf{DFT}_2 \to \mathsf{F}_2$$
$$\mathbf{DCT\text{-}2}_2 \to \mathrm{diag}(1, 1/\sqrt{2})\, \mathsf{F}_2$$
$$\mathbf{DCT\text{-}4}_2 \to \mathsf{J}_2\, \mathsf{R}_{13\pi/8}$$

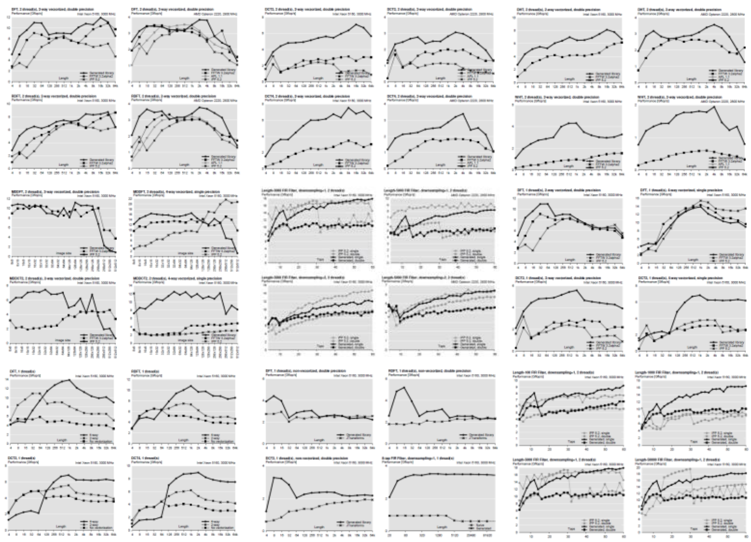
Generating 100s of FFTWs

PhD thesis Voronenko, 2009

Transform	Code size non-parallelized	Code size parallelized
no vectorization		
DFT	13.1 KLOC / 0.59 MB	10.3 KLOC / 0.45 MB
RDFT	8.5 KLOC / 0.36 MB	8.8 KLOC / 0.39 MB
DHT	9.1 KLOC / 0.40 MB	9.4 KLOC / 0.39 MB
DCT-2	12.0 KLOC / 0.55 MB	12.4 KLOC / 0.57 MB
DCT-3	12.0 KLOC / 0.56 MB	12.3 KLOC / 0.59 MB
DCT-4	6.8 KLOC / 0.33 MB	7.1 KLOC / 0.35 MB
WHT	5.6 KLOC / 0.21 MB	—
2-way vectorization		
DFT	14.8 KLOC / 0.73 MB	15.0 KLOC / 0.74 MB
RDFT	15.6 KLOC / 0.76 MB	16.0 KLOC / 0.81 MB
scaled RDFT	16.0 KLOC / 0.78 MB	—
DHT	16.9 KLOC / 0.83 MB	17.2 KLOC / 0.87 MB
DCT-2	20.7 KLOC / 1.10 MB	21.0 KLOC / 1.09 MB
DCT-3	27.9 KLOC / 1.56 MB	28.2 KLOC / 1.59 MB
DCT-4	7.8 KLOC / 0.47 MB	8.1 KLOC / 0.50 MB
WHT	6.9 KLOC / 0.32 MB	5.8 KLOC / 0.26 MB
FIR Filter	167 KLOC / 7.75 MB	120 KLOC / 5.12 MB
Downsampled FIR Filter	100 KLOC / 4.2 MB	68 KLOC / 2.76 MB
4-way vectorization		
DFT	17.9 KLOC / 1.09 MB	18.2 KLOC / 1.11 MB
RDFT	16.2 KLOC / 0.86 MB	16.5 KLOC / 0.91 MB
scaled RDFT	16.5 KLOC / 0.88 MB	—
DHT	17.9 KLOC / 1.02 MB	18.3 KLOC / 1.04 MB
DCT-2	23.3 KLOC / 1.50 MB	23.6 KLOC / 1.53 MB
DCT-3	32.0 KLOC / 2.17 MB	32.3 KLOC / 2.20 MB
DCT-4	8.3 KLOC / 0.63 MB	8.6 KLOC / 0.66 MB
WHT	8.5 KLOC / 0.53 MB	6.9 KLOC / 0.4 MB
2D DFT	20.6 KLOC / 1.32 MB	20.8 KLOC / 1.33 MB
2D DCT-2	27.0 KLOC / 2.1 MB	27.2 KLOC / 2.11 MB
FIR Filter	109 KLOC / 5.69 MB	74 KLOC / 3.44 MB
Downsampled FIR Filter	151 KLOC / 7.7 MB	92 KLOC / 4.61 MB

Generating 100s of FFTWs

PhD thesis Voronenko, 2009



Computer generated Functions for Intel IPP 6.0

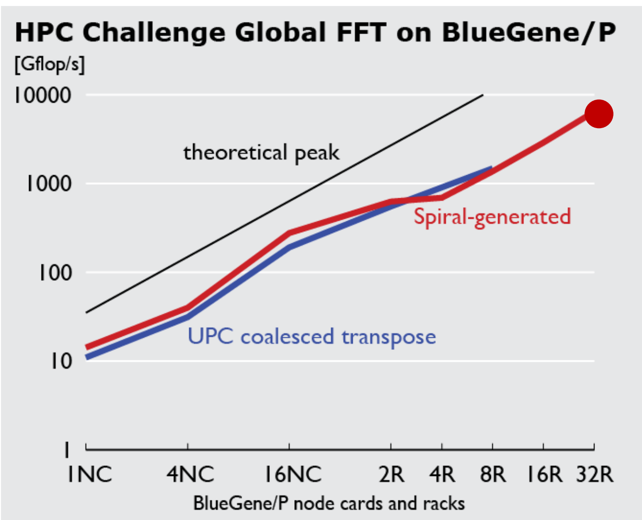
3984 C functions
1M lines of code

- *Transforms: DFT (fwd+inv), RDFT (fwd+inv), DCT2, DCT3, DCT4, DHT, WHT*
- *Sizes: 2–64 (DFT, RDFT, DHT); 2-powers (DCTs, WHT)*
- *Precision: single, double*
- *Data type: scalar, SSE, AVX (DFT, DCT), LRB (DFT)*

Computer generated

Results: SpiralGen Inc.

Very Large Scale: BG/P



2010 HPC Challenge Class I Award, Almasi et al.

6.4 Tflop/s

32 racks
= 32K node cards
= 128K cores

Organization

- Spiral: Basic system
- Vectorization
- General input size
- Results
- ***Final remarks***

Spiral: Summary

- **Spiral:**
  Successful approach to automating the development of computing software

  Commercial proof-of-concept

$$\mathrm{DFT}_{64}$$

```
void dft64(float  *Y, float  *X) {
  __m512 U912, U913, U914, U915,...
  __m512  *a2153, *a2155;
  a2153 = ((__m512  *) X);  s1107 = *(a2153);
  s1108 = *((a2153 + 4));   t1323 = _mm512_add_ps(s1107,s1108);
  t1324 = _mm512_sub_ps(s1107,s1108);
    <many more lines>
  U926 = _mm512_swizupconv_r32(…);
  s1121 = _mm512_madd231_ps(_mm512_mul_ps(_mm512_mask_or_pi(
    _mm512_set_1to16_ps(0.70710678118654757),0xAAAA,a2154,U926),t1341),
    _mm512_mask_sub_ps(_mm512_set_1to16_ps(0.70710678118654757),…),
    _mm512_swizupconv_r32(t1341,_MM_SWIZ_REG_CDAB));
  U927 = _mm512_swizupconv_r32
    <many more lines>
}
```

- **Key ideas:**
  Algorithm knowledge:
  Domain specific symbolic representation

  $$\mathrm{DFT}_4 \rightarrow (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\mathrm{T}^4_2(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathrm{L}^4_2$$

  Platform knowledge:
  Tagged rewrite rules, SIMD specification

  $$\underbrace{\mathrm{I}_m \otimes A_n}_{\mathsf{smp}(p,\mu)} \rightarrow \mathrm{I}_p \otimes_{\|} \left(\mathrm{I}_{m/p} \otimes A_n\right)$$

Glimpse of other topics …

LGen: Generator for Basic Linear Algebra

Spampinato & P, CGO 2014

BLAC $y = x^T(A+B)y + \delta$

Algorithm: Tiling decision and propagation
(LL) $\left[y = x^T(A+B)y + \delta\right]_{2,3}$

vectorization

Algorithm
(Σ-LL) $\sum_{i,j,i',j'} S_i S_{i'} \left(G_{i'} G_i A G_j G_{j'}\right) \left(G_{j'} G_j x\right) \dots$

locality optimization

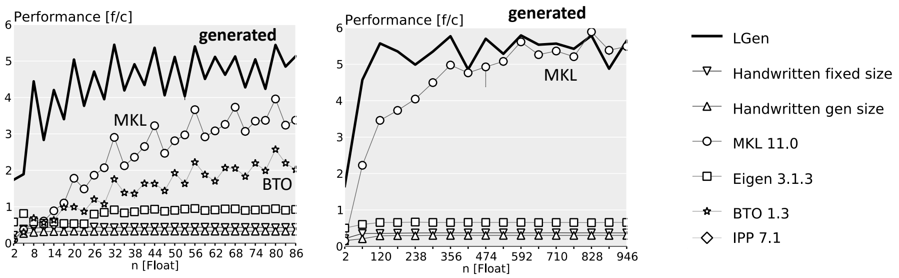
C Program

```
void kernel(float *x, float *A, float *B, …) {
float t0_64_0, t0_64_1, t0_64_2, t0_64_3 …;
    t0_57_0 = A[0];
    t0_56_0 = A[1];
    …
    t0_59_0 = t0_57_0 + t0_33_0;
    t0_63_0 = t0_59_0 * t0_9_0;
    t0_59_1 = t0_56_0 + t0_32_0;
    t0_60_0 = t0_59_1 * t0_8_0;
    < many more lines>
```

code style
code level optimization

LGen: Sample Results

$C = \alpha AB + \beta C$ $\quad\quad$ $C = \alpha(A_0 + A_1)^T B + \beta C$

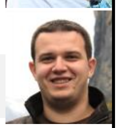


$A \in \mathbb{R}^{n\times 4}$ $\quad\quad$ $A_0 \in \mathbb{R}^{4\times 4}$

$B \in \mathbb{R}^{4\times n}$ $\quad\quad$ $B \in \mathbb{R}^{4\times n}$

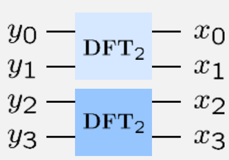
PL Support: Example Code Style

Ofenbeck, Rompf, Stojanov, Odersky & P, GPCE 2012



SPL $\quad y = (\mathrm{I}_2 \otimes \mathbf{DFT}_2)x$

Data flow graph



Scala function

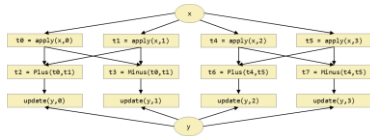
```
def f(x: Array[Double], y: Array[Double]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```

```
def f(x: Array[Rep[Double]],
      y: Array[Rep[Double]]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```

scalarized

```
t0 = s0 + s1;
t1 = s0 - s1;
t2 = s2 + s3;
t2 = s2 - s3;
```

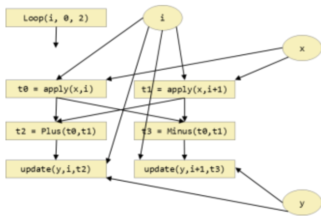
```
def f(x: Rep[Array[Double]],
      y: Rep[Array[Double]]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```



unrolled, scalar repl.

```
t0 = x[0];
t1 = x[1];
t2 = t0 + t1;
y[0] = t2;
t3 = t0 - t1;
y[1] = t3;
t4 = x[0];
t5 = x[1];
t6 = t4 + x5;
y[0] = t6;
t7 = t4 - x5;
y[3] = t7;
```

```
def f(x: Rep[Array[Double]],
      y: Rep[Array[Double]]) = {
      for (i <- 0 until 2: Rep[Range]) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```



looped, scalar repl.

```
for (int i=0; i < 2; i++)
{
     t0 = x[i];
     t1 = x[i+1];
     t2 = t0 + t1;
     y[i] = t2;
     t3 = t0 - t1;
     y[i+1] = t3;
}
```

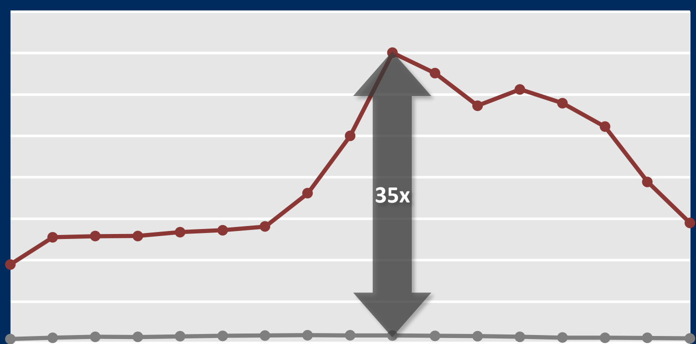
Staging enables program generation

Abstracting over code style = abstracting over staging decisions

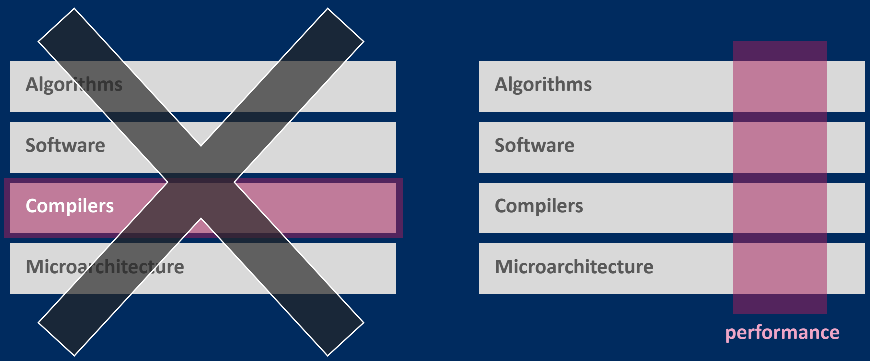
```
def f[L[_],A[_],T](looptype: L, x: A[Array[T]], y: A[Array[T]]) = {
      for (i <- 0 until 2: L[Range]) {
        y(2*i)  = x(i*2) + x(i*2+1)
        y(2*i+1)= x(i*2) - x(i*2+1)
      }
}
```

Advanced Systems Lab

Conclusions



Straightforward implementations often underperform by an order of magnitude, even if single-threaded



Performance is different than other software quality features

Research Questions

- ***How to port performance?***
- **How to automate the production of fastest numerical code?**
  - *Domain-specific languages*
  - *Rewriting*
  - *Compilers*
  - *Machine Learning*
- **What program language features help with program generation?**
- **What environment should be used to build generators?**
- **How to represent mathematical functionality?**
- **How to formalize the mapping to fast code?**
- **How to handle various forms of parallelism?**
- **How to integrate into standard work flows?**