How to Write Fast Numerical Code

Spring 2019

Lecture: Memory hierarchy, locality, caches

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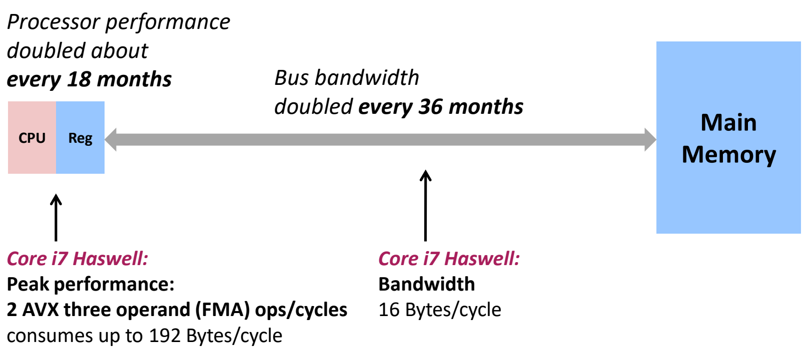
Organization

- **Temporal and spatial locality**
- **Memory hierarchy**
- **Caches**

*Chapter 5 in **Computer Systems: A Programmer's Perspective**, 2nd edition, Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010*

Part of these slides are adapted from the course associated with this book

Problem: Processor-Memory Bottleneck



Solution: Caches/Memory hierarchy

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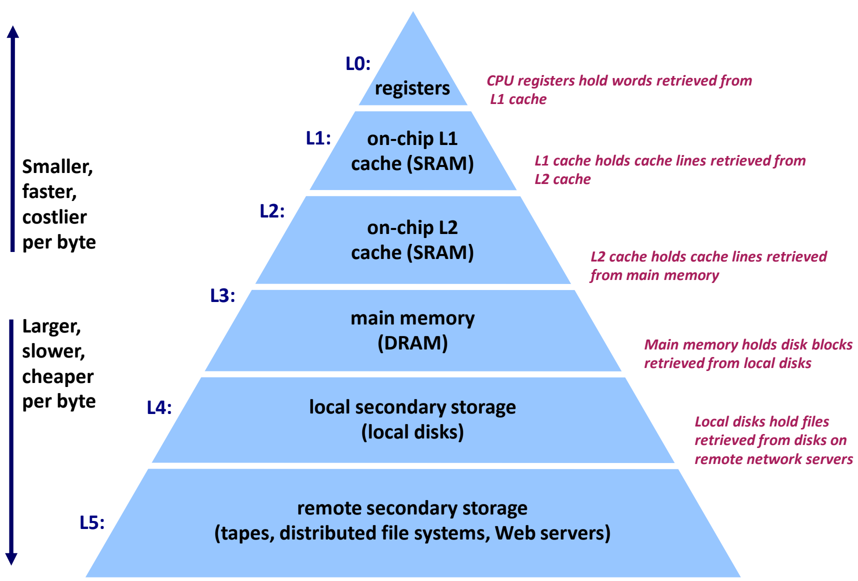
Typical Memory Hierarchy



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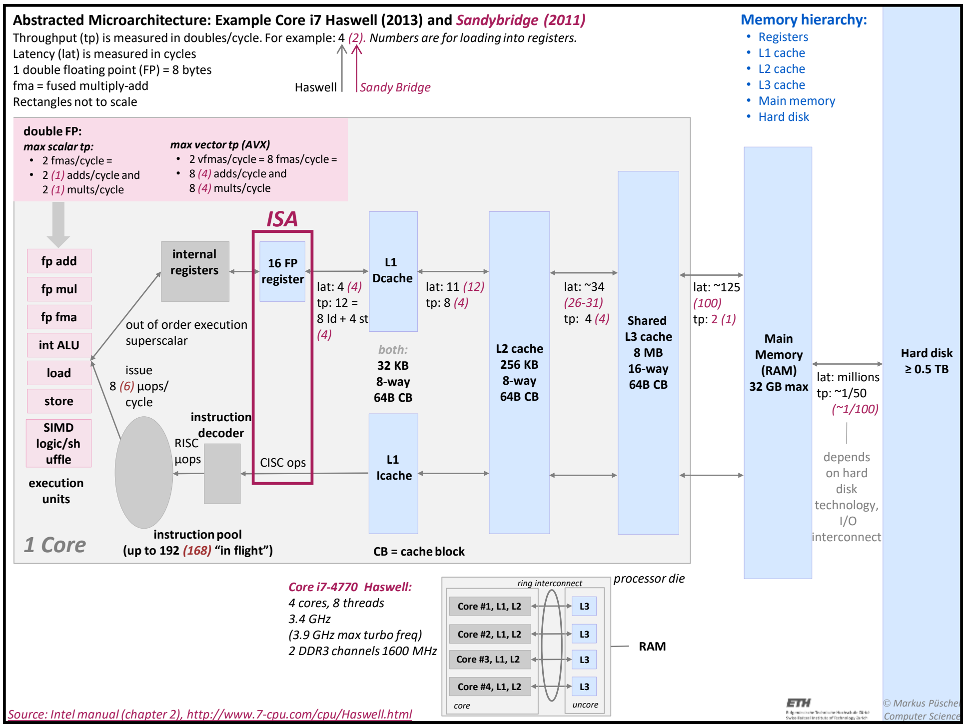
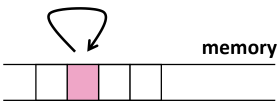


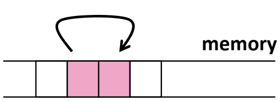
Why Caches Work: Locality

- ***Locality:*** **Programs tend to use data and instructions with addresses near or equal to those they have used recently**
  History of locality

- ***Temporal locality:***
  Recently referenced items are likely to be referenced again in the near future



- ***Spatial locality:***
  Items with nearby addresses tend to be referenced close together in time



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Example: Locality?

```
sum = 0;
for (i = 0; i < n; i++)
  sum += a[i];
return sum;
```

- **Data:**
  - Temporal: **sum** referenced in each iteration
  - Spatial: array **a[]** accessed consecutively
- **Instructions:**
  - Temporal: loops cycle through the same instructions
  - Spatial: instructions referenced in sequence

- ***Being able to assess the locality of code is a crucial skill for a performance programmer***

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Locality Example #1

```
int sum_array_rows(double a[M][N])
{
  int i, j, sum = 0;

  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      sum += a[i][j];
  return sum;
}
```

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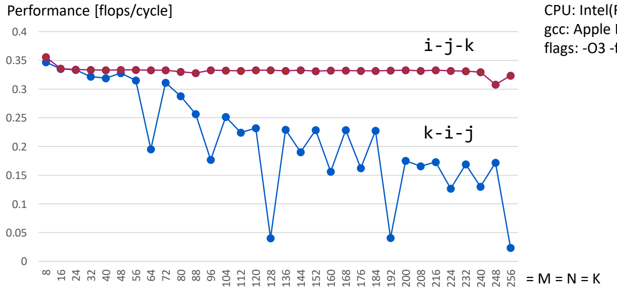
Locality Example #2

```
int sum_array_cols(double a[M][N])
{
  int i, j, sum = 0;

  for (j = 0; j < N; j++)
    for (i = 0; i < M; i++)
      sum += a[i][j];
  return sum;
}
```


Locality Example #3

```
int sum_array_3d(double a[M][N][K])
{
  int i, j, k, sum = 0;

  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      for (k = 0; k < K; k++)
        sum += a[k][i][j];
  return sum;
}
```

How to improve locality?



CPU: Intel(R) Core(TM) i7-4980HQ CPU @ 2.80GHz
gcc: Apple LLVM version 8.0.0 (clang-800.0.42.1)
flags: -O3 -fno-vectorize

Operational Intensity Again

- **Definition: Given a program P, assume cold (empty) cache**

Operational intensity: $I(n) = \frac{W(n)}{Q(n)}$

W(n): #flops (input size n)

Q(n): #bytes transferred cache ↔ memory (for input size n)

- **Examples: Determine asymptotic bounds on I(n)**
  - Vector sum: y = x + y **O(1)**
  - Matrix-vector product: y = Ax **O(1)**
  - Fast Fourier transform **O(log(n))**
  - Matrix-matrix product: C = AB + C **O(n)**

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Compute/Memory Bound

- **A function/piece of code is:**
  - ***Compute bound*** if it has high operational intensity
  - ***Memory bound*** if it has low operational intensity
- **Relationship between operational intensity and locality?**
  - They are closely related
  - Operational intensity only describes the boundary last level cache/memory

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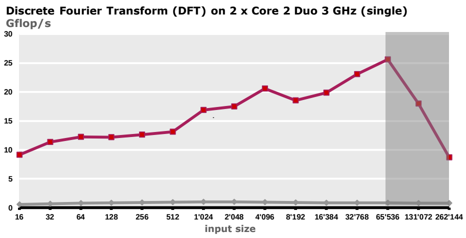
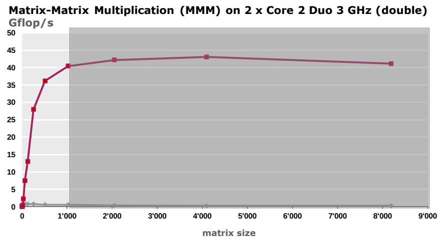
Effects

FFT: *I(n) = O(log(n))*



MMM: *I(n) = O(n)*



Up to 40-50% peak
Performance drop outside last level cache (LLC)
Most time spent transferring data

Up to 80-90% peak
Performance can be maintained outside LLC
Cache miss time compensated/hidden by computation

Cache

- ***Definition:*** **Computer memory with short access time used for the storage of frequently or recently used instructions or data**

- **Naturally supports *temporal locality***
- ***Spatial locality* is supported by transferring data in blocks**
  - Core family: one block = 64 B = 8 doubles

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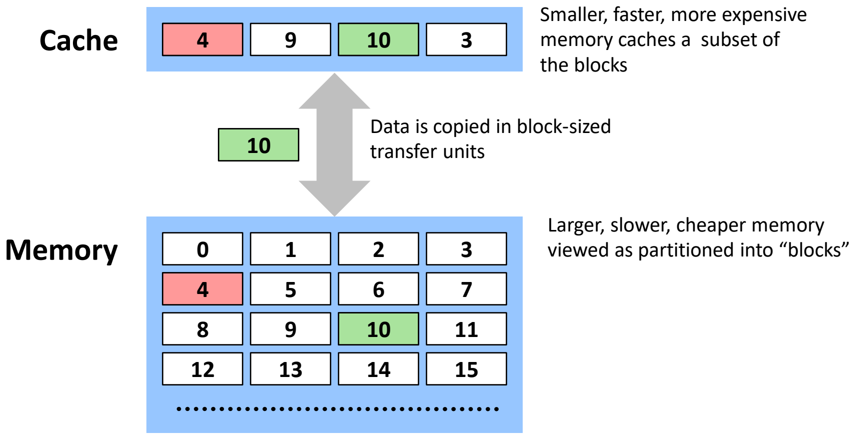
General Cache Mechanics



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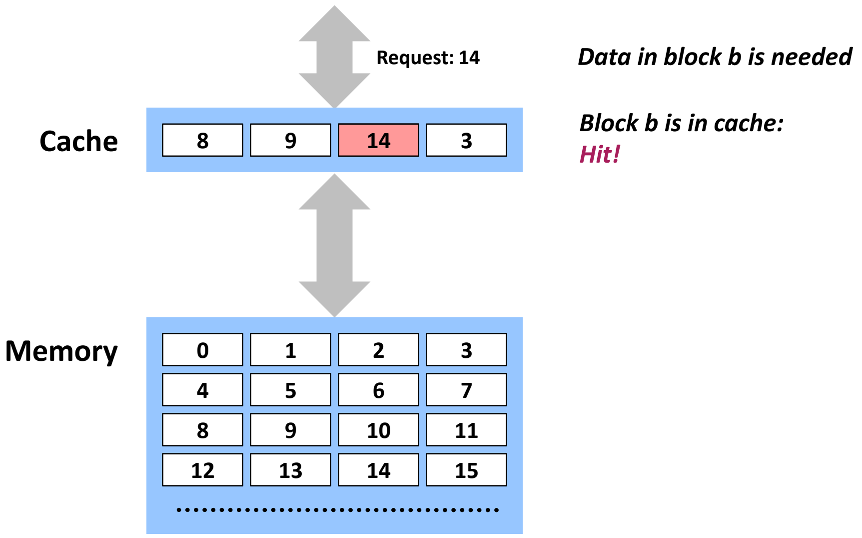
General Cache Concepts: Hit



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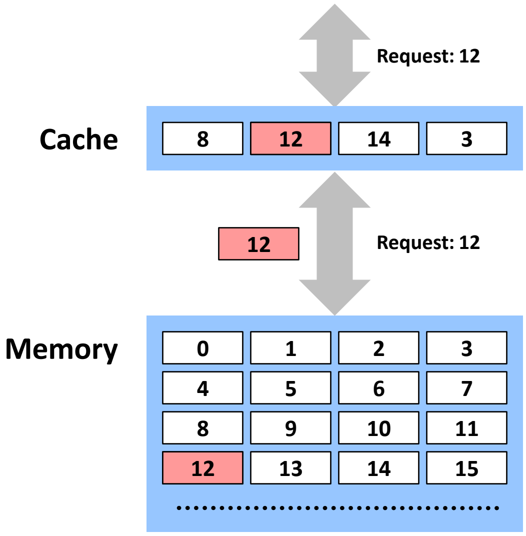
General Cache Concepts: Miss



Data in block b is needed

Block b is not in cache:
Miss!

Block b is fetched from memory

Block b is stored in cache
- *Placement policy:* determines where b goes
- *Replacement policy:* determines which block gets evicted (victim)

Types of Cache Misses (The 3 C's)

- ***Compulsory (cold)* miss**
  Occurs on first access to a block
- ***Capacity* miss**
  Occurs when working set is larger than the cache
- ***Conflict* miss**
  Conflict misses occur when the cache is large enough, but multiple data objects all map to the same slot

- **Not a clean classification but still useful**

Cache Structure

- Draw a direct mapped cache (E = 1, B = 4 doubles, S = 8)
- Show how blocks are mapped into cache

Example (S=8, E=1)

Ignore the variables sum, i, j

```
int sum_array_rows(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (i = 0; i < 16; i++)
    for (j = 0; j < 16; j++)
      sum += a[i][j];
  return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (j = 0; j < 16; j++)
    for (i = 0; i < 16; i++)
      sum += a[i][j];
  return sum;
}
```

assume: cold (empty) cache, a[0][0] goes here

B = 32 byte = 4 doubles

blackboard

Cache Structure

- Add associativity (E = 2, B = 4 doubles, S = 8)
- Show how elements are mapped into cache

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Example (S=4, E=2)

Ignore the variables sum, i, j

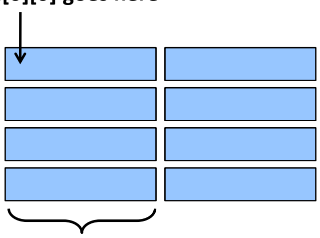
```
int sum_array_rows(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (i = 0; i < 16; i++)
    for (j = 0; j < 16; j++)
      sum += a[i][j];
  return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
  int i, j;
  double sum = 0;

  for (j = 0; j < 16; j++)
    for (i = 0; i < 16; i++)
      sum += a[i][j];
  return sum;
}
```

assume: cold (empty) cache,
a[0][0] goes here



B = 32 byte = 4 doubles

blackboard

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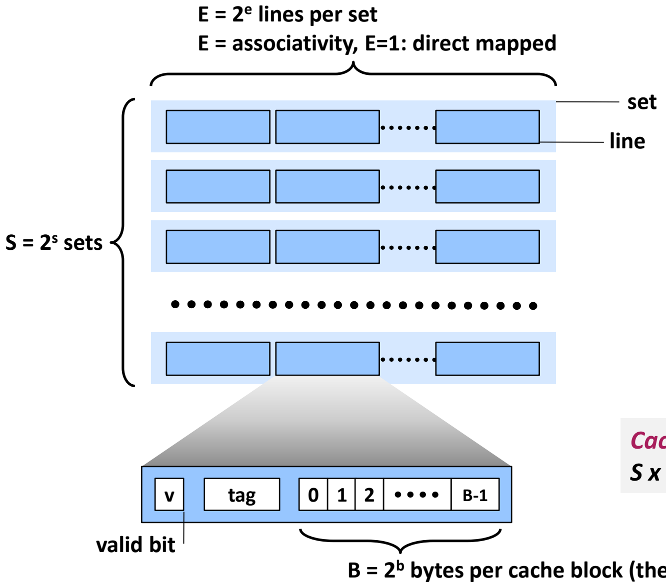
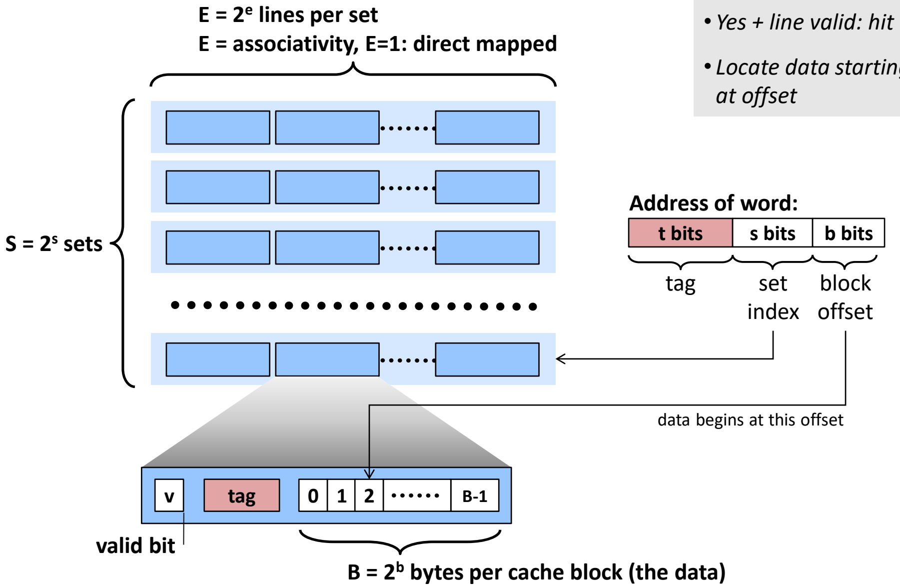
General Cache Organization (S, E, B)



Cache size:
S x E x B data bytes

Cache Read

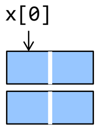
- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



Terminology

- **Direct mapped cache:**
  - Cache with E = 1
  - Means every block from memory has a unique location in cache
- **Fully associative cache**
  - Cache with S = 1 (i.e., maximal E)
  - Means every block from memory can be mapped to any location in cache
  - In practice to expensive to build
  - One can view the register file as a fully associative cache
- **LRU (least recently used) replacement**
  - when selecting which block should be replaced (happens only for E > 1), the least recently used one is chosen

Small Example, Part 1



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

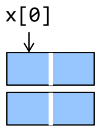
Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for i = 0:7
    access(x[i])
```

Access pattern: `0123456701234567`
Hit/Miss: `MHMHMHMHMHMHMHMH`

Result: 8 misses, 8 hits
Spatial locality: yes
Temporal locality: no

Small Example, Part 2



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

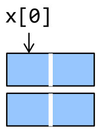
Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for i = 0:2:7
    access(x[i])
  for i = 1:2:7
    access(x[i])
```

Access pattern: `0246135702461357`
Hit/Miss: `MMMMMMMMMMMMMMMM`

Result: 16 misses
Spatial locality: no
Temporal locality: no

Small Example, Part 3



Cache:
E = 1 (direct mapped)
S = 2
B = 16 (2 doubles)

Array (accessed twice in example)
`x = x[0], …, x[7]`

```
% Matlab style code
for j = 0:1
  for k = 0:1
    for i = 0:3
      access(x[i+4j])
```

Access pattern: `0123012345674567`
Hit/Miss: `MHMHHHHHMHMHHHHH`

Result: 4 misses, 12 hits (is optimal, why?)
Spatial locality: yes
Temporal locality: yes

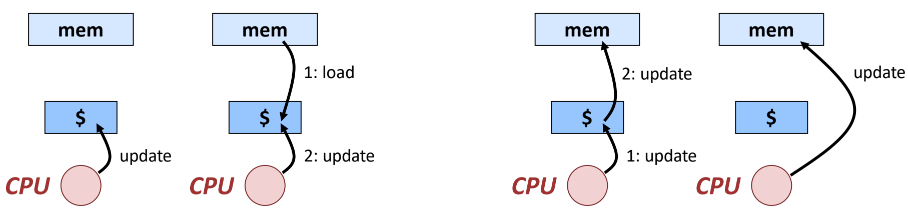
Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache: misses / accesses
    = 1 – hit rate
- **Hit Time**
  - Time to deliver a block in the cache to the processor
  - Core 2:
    3 clock cycles for L1
    14 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
  - Core 2: about 100 cycles for L2 miss

What about writes?

- **What to do on a write-hit?**
  - ***Write-through:*** write immediately to memory
  - ***Write-back:*** defer write to memory until replacement of line
- **What to do on a write-miss?**
  - ***Write-allocate:*** load into cache, update line in cache
  - ***No-write-allocate:*** writes immediately to memory

Write-back/write-allocate (Core) | ***Write-through/no-write-allocate***



Write-hit | **Write-miss** | **Write-hit** | **Write-miss**

Example: (Blackboard)

- z = x + y, x, y, z vector of length n
- assume they fit jointly in cache + cold cache
- memory traffic Q(n)?
- operational intensity I(n)?

Locality Optimization: Blocking

- Example: MMM (blackboard)

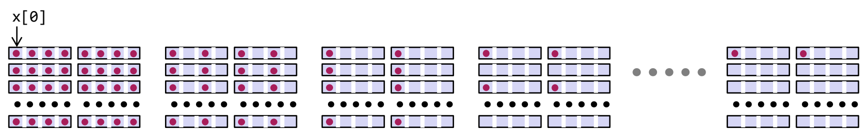
The Killer: Two-Power Strided Working Sets

```
% t = 1,2,4,8,… a 2-power
% size W of working set: W = n/t
for (i = 0; i < n; i += t)
  access(x[i])
for (i = 0; i < n; i += t)
  access(x[i])
```

blackboard

Cache: E = 2, B = 4 doubles

t = 1: | **t = 2:** | **t = 4:** | **t = 8:** | **t ≥ 4S:**



t = 1	t = 2	t = 4	t = 8	t ≥ 4S
Spatial locality Temporal locality: if W ≤ C	Some spatial locality Temporal locality: if W ≤ C/2	No spatial locality Temporal locality: if W ≤ C/4	No spatial locality Temporal locality: if W ≤ C/8	No spatial locality Temporal locality: if W ≤ 2

The Killer: Where Can It Occur?

- **Accessing two-power size 2D arrays (e.g., images) columnwise**
  - 2d Transforms
  - Stencil computations
  - Correlations
- **Various transform algorithms**
  - Fast Fourier transform
  - Wavelet transforms
  - Filter banks

Summary

- **It is important to assess temporal and spatial locality in the code**
- **Cache structure is determined by three parameters**
  - block size
  - number of sets
  - associativity
- **You should be able to roughly simulate a computation on paper**
- **Blocking to improve locality**
- **Two-power strides are problematic (conflict misses)**