How to Write Fast Numerical Code

Spring 2019, Lecture 1

Instructor: *Markus Püschel*

TAs: *Tyler Smith, Gagandeep Singh, Alen Stojanov*

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Picture: www.tapety-na-pulpit.org

Minds open…

… Laptops closed

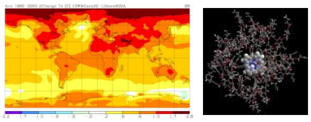
slide by Bertrand Meyer

2

Today

- **Motivation for this course**
- **Organization of this course**

Scientific Computing



Physics/biology simulations

Consumer Computing

Audio/image/video processing

Embedded Computing

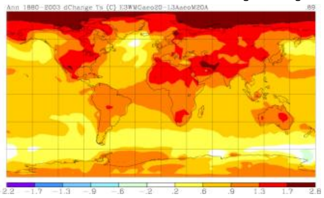
Signal processing, communication, control

Computing

- **Unlimited need for performance**
- **Large set of applications, but …**
- **Relatively small set of critical components (100s to 1000s)**
  - Matrix multiplication
  - Discrete Fourier transform (DFT)
  - Viterbi decoder
  - Shortest path computation
  - Stencils
  - Solving linear system
  - ....

Scientific Computing (Clusters/Supercomputers)

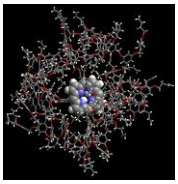
data.giss.nasa.gov



Climate modelling

Finance simulations

www.foresight.org



Molecular dynamics

Other application areas:
- Fluid dynamics
- Chemistry
- Biology
- Medicine
- Geophysics

Methods:
- Mostly linear algebra
- PDE solving
- Linear system solving
- Finite element methods
- Others

Consumer Computing (Desktop, Phone, …)

Photo/video processing

Audio coding

Security

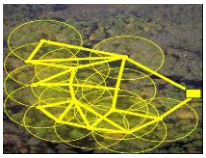
Original JPEG JPEG2000

Image compression

Methods:
- Linear algebra
- Transforms
- Filters
- Others

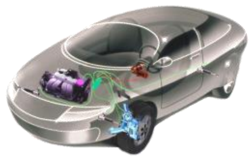
Embedded Computing (Low-Power Processors)

www.dei.unipd.it



Sensor networks

www.ece.drexel.edu



Cars

www.microway.com.au

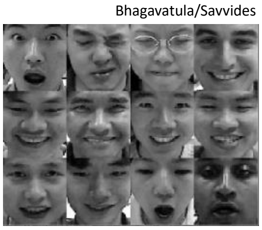
Robotics

Computation needed:
- Signal processing
- Control
- Communication

Methods:
- Linear algebra
- Transforms, Filters
- Coding

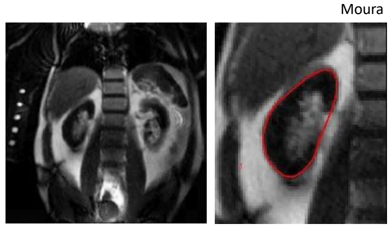
Research (Examples from Carnegie Mellon)

Bhagavatula/Savvides



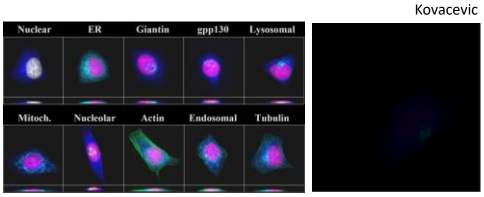
Biometrics

Moura



Medical Imaging

Kovacevic



Bioimaging

Kanade

Computer vision

Classes of Performance-Critical Functions

- Transforms
- Filters/correlation/convolution/stencils/interpolators
- Dense linear algebra functions
- Sparse linear algebra functions
- Coder/decoders
- Graph algorithms
- *… several others*

See also the 13 dwarfs/motifs in
http://www.eecs.berkeley.edu/Pubs/TechRpts/2006/EECS-2006-183.pdf

How Hard Is It to Get Fast Code?

- Algorithms
- Software
- Compilers
- Microarchitecture

"compute Fourier transform"

↓

"fast Fourier transform"
O(nlog(n)) or 4nlog(n) + 3n

↓

e.g., a C function

↓

optimized executable

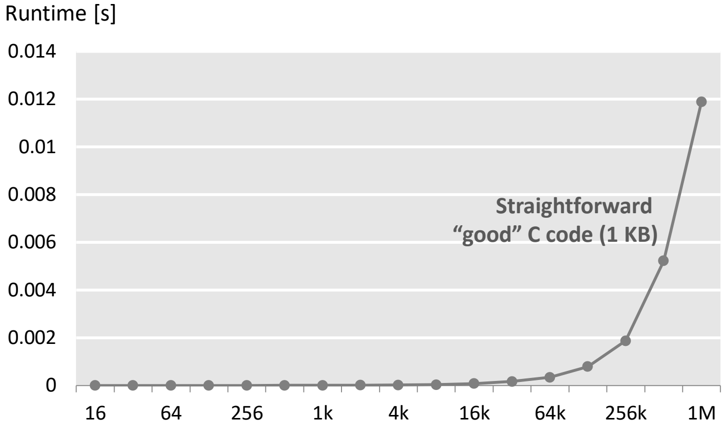
↓

high performance

How well does this work?

The Problem: Example 1

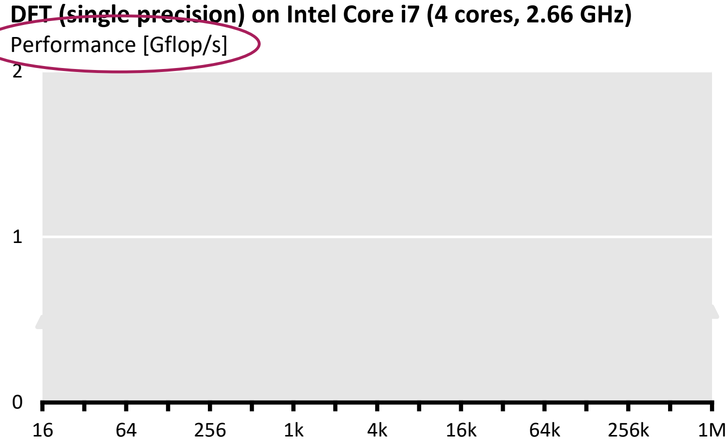
DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)



 **or**  **?**

The Problem: Example 1

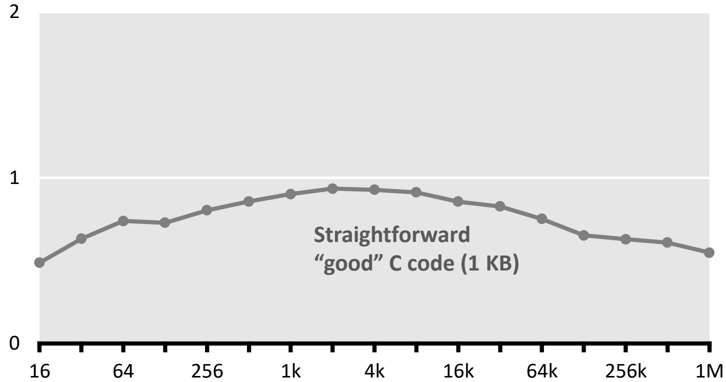
DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)



The Problem: Example 1

DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)

Performance [Gflop/s]



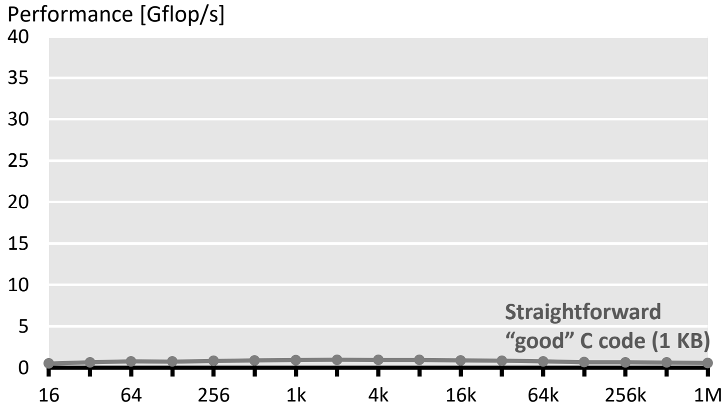
 **or**  **?**

13

The Problem: Example 1

DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)

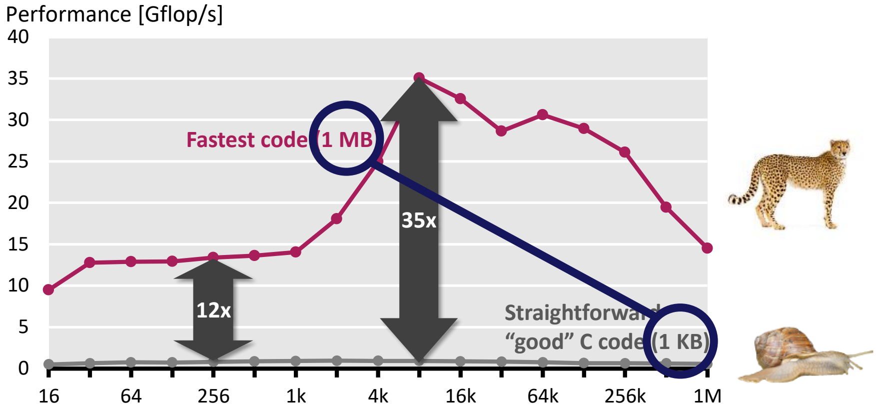
Performance [Gflop/s]



14

The Problem: Example 1

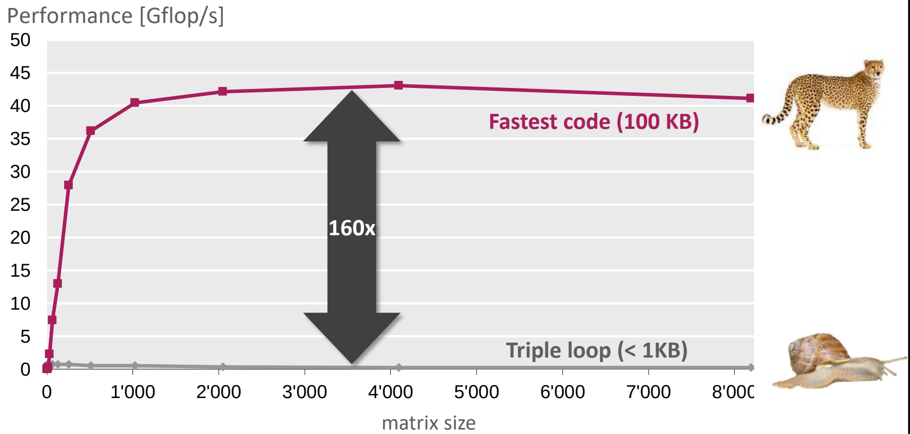
DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)



- Vendor compiler, best flags
- Roughly same operations count

The Problem: Example 2

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



- Vendor compiler, best flags
- Exact same operations count ($2n^3$)

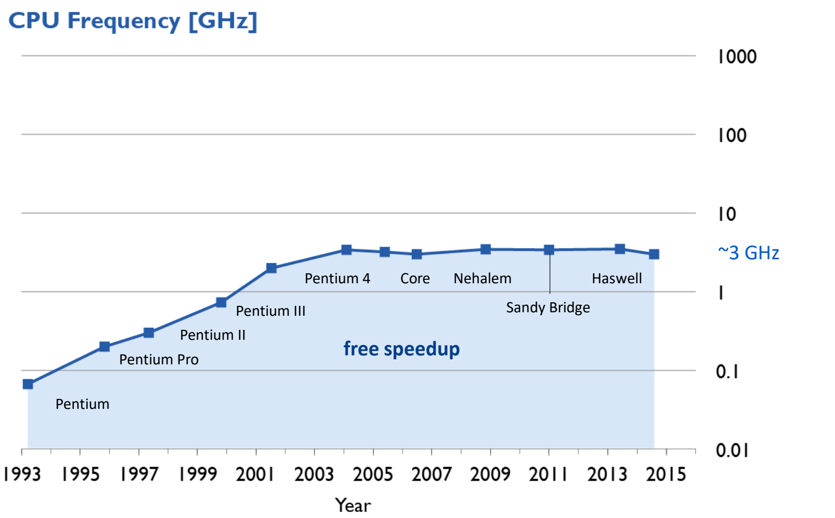
Model predictive control	Singular-value decomposition
Eigenvalues	Mean shift algorithm for segmentation
LU factorization	Stencil computations
Optimal binary search organization	Displacement based algorithms
Image color conversions	Motion estimation
Image geometry transformations	Multiresolution classifier
Enclosing ball of points	Kalman filter
Metropolis algorithm, Monte Carlo	Object detection
Seam carving	IIR filters
SURF feature detection	Arithmetic for large numbers
Submodular function optimization	Optimal binary search organization
Graph cuts, Edmond-Karps Algorithm	Software defined radio
Gaussian filter	Shortest path problem
Black Scholes option pricing	Feature set for biomedical imaging
Disparity map refinement	Biometrics identification

"Theorem:"

Let f be a mathematical function to be implemented on a state-of-the-art processor. Then

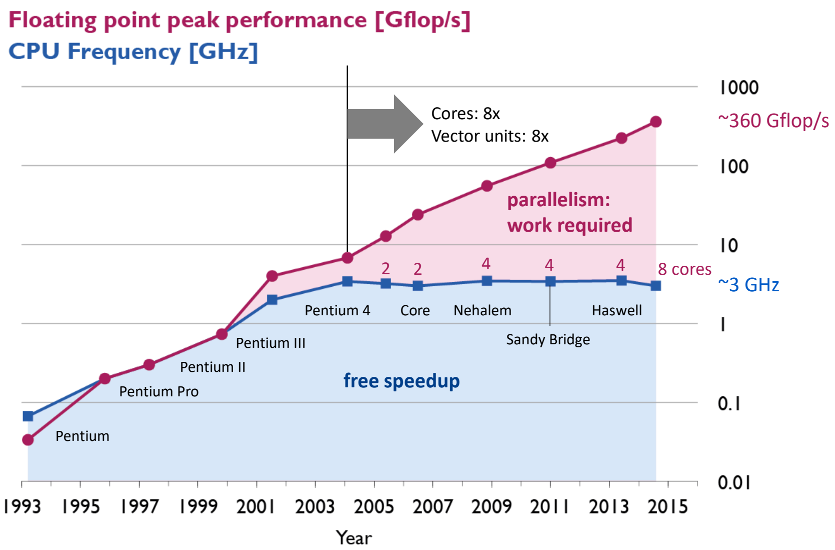
$$\frac{\textit{Performance of optimal implementation of } f}{\textit{Performance of straightforward implementation of } f} \approx \textit{10–100}$$

Evolutions of Processors (Intel)



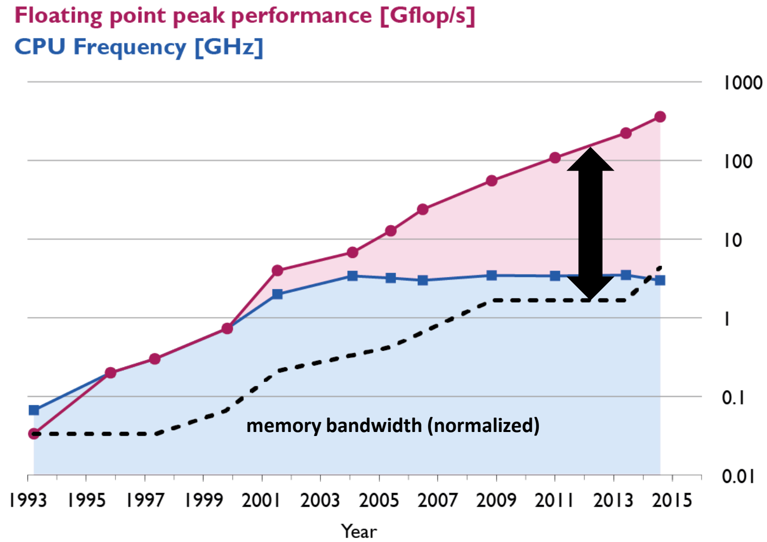
Source: Wikipedia/Intel/PCGuide

Evolutions of Processors (Intel)



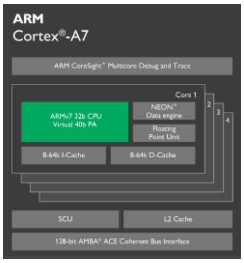
Source: Wikipedia/Intel/PCGuide

Evolutions of Processors (Intel)



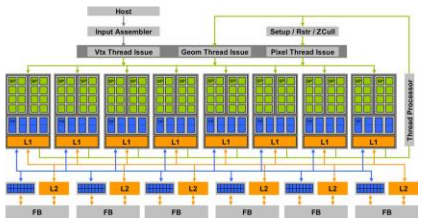
Source: Wikipedia/Intel/PCGuide

And there is Processor Variety …



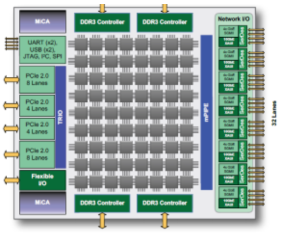
arm.com

Nvidia Tesla



beyond3d.com

Domain-specific (here: Tile)



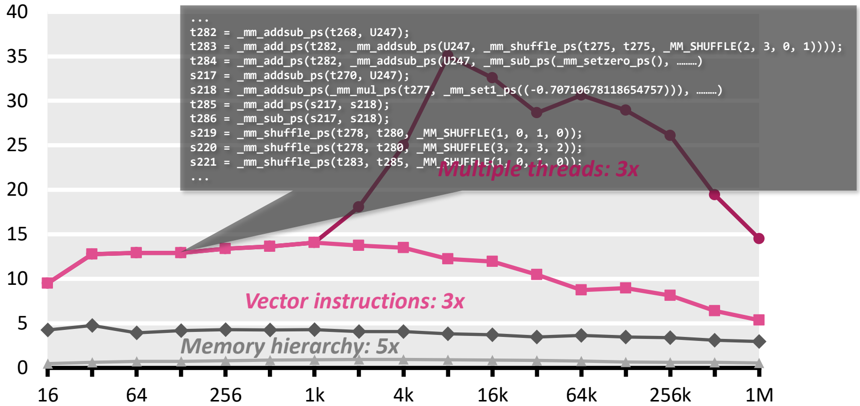
mellanox.com

FPGA accelerators

nallatech.com

DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)

Performance [Gflop/s]

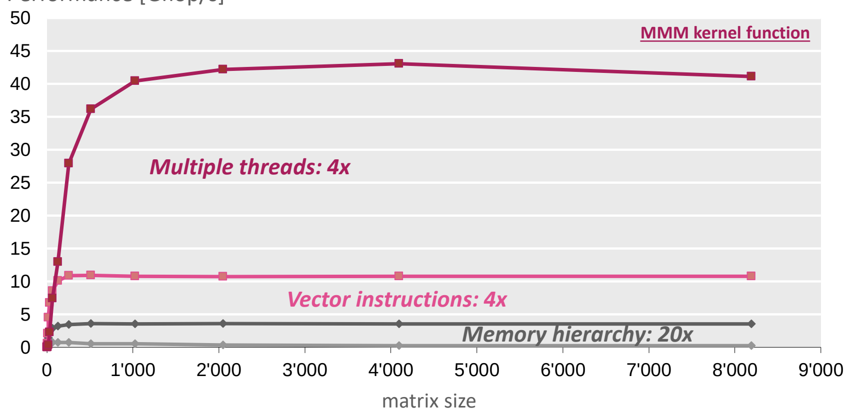



```
...
t282 = _mm_addsub_ps(t268, U247);
t283 = _mm_add_ps(t282, _mm_addsub_ps(U247, _mm_shuffle_ps(t275, t275, _MM_SHUFFLE(2, 3, 0, 1))));
t284 = _mm_add_ps(t282, _mm_addsub_ps(U247, _mm_sub_ps(_mm_setzero_ps(), ………)
s217 = _mm_addsub_ps(t270, U247);
s218 = _mm_addsub_ps(_mm_mul_ps(t277, _mm_set1_ps((-0.7071067811865475 7))), ………)
t285 = _mm_add_ps(s217, s218);
t286 = _mm_sub_ps(s217, s218);
s219 = _mm_shuffle_ps(t278, t280, _MM_SHUFFLE(1, 0, 1, 0));
s220 = _mm_shuffle_ps(t278, t280, _MM_SHUFFLE(3, 2, 3, 2));
s221 = _mm_shuffle_ps(t283, t285, _MM_SHUFFLE(1, 0, 1, 0));
...
```

- Compiler doesn't do the job
- Doing by hand: ***nightmare***

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz

Performance [Gflop/s]



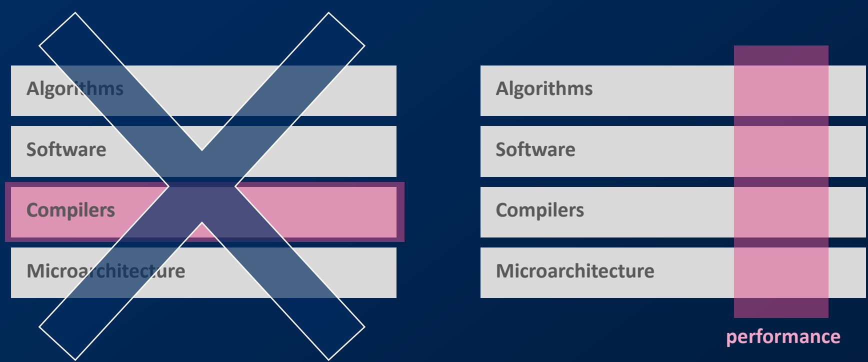
- Compiler doesn't do the job
- Doing by hand: ***nightmare***

Summary and Facts I

- **Implementations with same operations count can have vastly different performance (up to 100x and more)**
  - A cache miss can be 100x more expensive than an operation
  - Vector instructions
  - Multiple cores = processors on one die
- **Minimizing operations count ≠ maximizing performance**
- **End of free speed-up for legacy code**
  - Future performance gains through increasing parallelism

Summary and Facts II

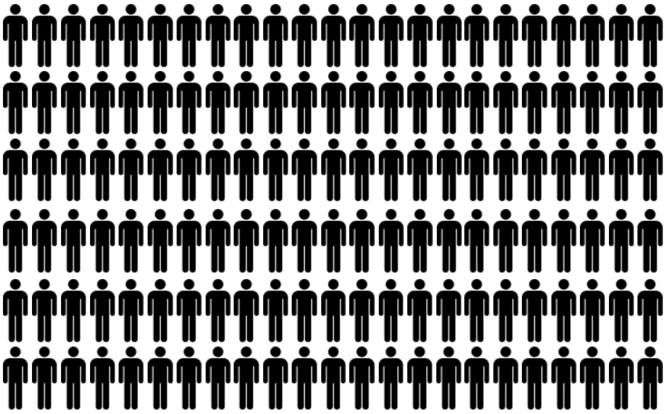
- **It is very difficult to write the fastest code**
  - Tuning for memory hierarchy
  - Vector instructions
  - Efficient parallelization (multiple threads)
  - Requires expert knowledge in algorithms, coding, and architecture
- **Fast code can be large**
  - Can violate "good" software engineering practices
- **Compilers often can't do the job**
  - Often intricate changes in the algorithm required
  - Parallelization/vectorization still unsolved
- **Highest performance is in general non-portable**



Performance is different than other software quality features

Performance/Productivity **Challenge**

Current Solution

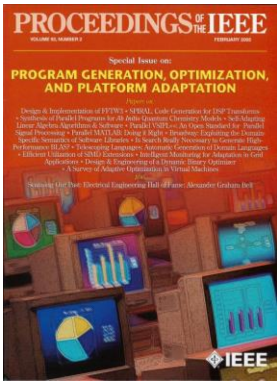


Legions of programmers implement and optimize the ***same*** functionality for ***every*** platform and ***whenever*** a new platform comes out

Better Solution: Autotuning

- **Automate (parts of) the implementation or optimization**

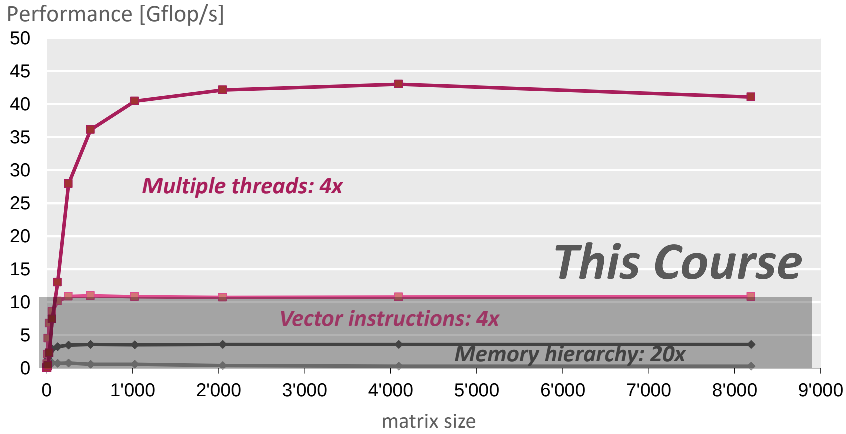
- **Research efforts**
  - Linear algebra: ***Phipac/ATLAS***, LAPACK, ***Sparsity/Bebop/OSKI***, Flame
  - Tensor computations
  - PDE/finite elements: Fenics
  - Adaptive sorting
  - ***Fourier transform: FFTW***
  - Linear transforms: Spiral
  - …others
  - New compiler techniques



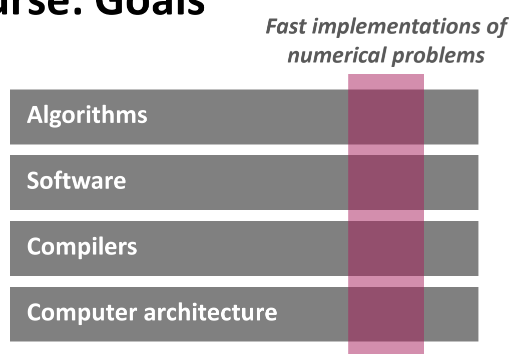
Proceedings of the IEEE special issue, Feb. 2005

Promising new area but much more work needed …

Matrix-Matrix Multiplication (MMM) on 2 x Core 2 Duo 3 GHz



This Course: Goals



- **Obtain an understanding of performance (runtime)**
- **Learn how to write *fast code* for numerical problems**
  - Focus: Memory hierarchy and vector instructions
  - Principles studied using important examples
  - *Applied in homeworks and a semester-long research project*
- **Learn about autotuning**

Today

- Motivation for this course
- **Organization of this course**

33

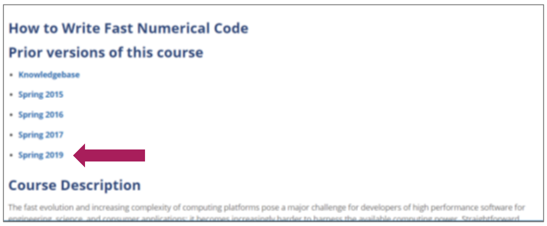
Course: Times and Places

- **Lectures:**
  - Monday 10-12, HG D3.2
  - Thursday 9-10, CAB G51
- **Recitations:**
  - Wednesday 13-15, HG D3.2
- **New course drop-out rule:** ***Deadline March 17th***

34

Course Website Has all Info

https://acl.inf.ethz.ch/teaching/fastcode/

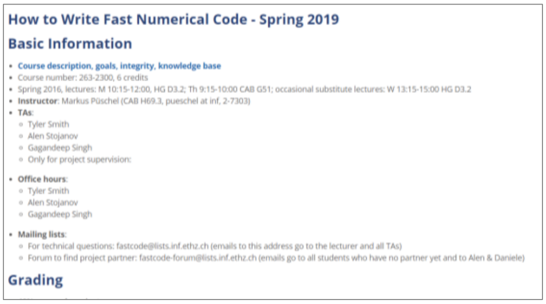

How to Write Fast Numerical Code

Prior versions of this course

- Knowledgebase
- Spring 2015
- Spring 2016
- Spring 2017
- Spring 2019

Course Description

The fast evolution and increasing complexity of computing platforms pose a major challenge for developers of high performance software for


How to Write Fast Numerical Code - Spring 2019

Basic Information

- Course description, goals, integrity, knowledge base
- Course number: 263-2300, 6 credits
- Spring 2016, lectures: M 10:15-12:00, HG D3.2; Th 9:15-10:00 CAB G51; occasional substitute lectures: W 13:15-15:00 HG D3.2
- **Instructor**: Markus Püschel (CAB H69.3, pueschel at inf, 2-7303)
- **TAs**:
  - Tyler Smith
  - Alen Stojanov
  - Gagandeep Singh
  - Only for project supervision:
- **Office hours**:
  - Tyler Smith
  - Alen Stojanov
  - Gagandeep Singh
- **Mailing lists**:
  - For technical questions: fastcode@lists.inf.ethz.ch (emails to this address go to the lecturer and all TAs)
  - Forum to find project partner: fastcode-forum@lists.inf.ethz.ch (emails go to all students who have no partner yet and to Alen & Daniele)

Grading

About this Course

- **Team**
  - TAs: Tyler Smith / Gagandeep Singh / Alen Stojanov

- **Office hours: to be determined**
- **Course website has *ALL* information**
- **Questions: fastcode@lists.inf.ethz.ch**
- **Finding project partner: fastcode-forum@lists.inf.ethz.ch**

About this Course (cont'd)

- **Requirements**
  - solid C programming skills
  - matrix algebra
  - Master student or above
- **Grading**
  - 40% research project
  - 25% midterm exam
  - 35% homework
- **Wednesday slot**
  - Gives you scheduled time to work together
  - Occasionally I will move lecture there (check course website)
  - By default will not take place

Research Project: Overview

- **Teams of 4**
- **Yes: 4**
- ***Topic:* Very fast implementation of a numerical problem**
- ***Until March 4th:***
  - *find a project team*
  - suggest to me a problem or I give you a problem
    Tip: pick something from your research or that you are interested in
  - *Register on project website + you get svn access*
- **Show "milestones" during semester**
- **One-on-one meetings**
- **Give short presentation end of semester**
- **Write 6 page standard conference paper (template will be provided)**
- **Submit final code (early semester break)**

Finding Project Team

- **Teams of 4: no exceptions**
- **Use fastcode-forum@lists.inf.ethz.ch:**
  - "I have a project (short description) and am looking for partners"
  - "I am looking for a team, am interested in anything related to visual computing"
- **In the beginning all of you are registered to that list**
- **Once team is formed (with or without project fixed) inform head TA, you will get deregistered from the list**

39

Finding Project

- **Pick something you are interested in**
- **Ok if prior code exists**
- **Nothing that is dominated by**
  - dense linear algebra computations (matrix-matrix mult, solving linear systems, Cholesky factorization etc.)
  - fast Fourier transform
- **Exact scope can be adapted during semester**
  - reduced to critical component
  - specialized
- ***You are in charge of your project!***
  - If too big, adapt
  - If too easy, expand
  - ***Don't come after 2 months and say project does not work***

40

Organize Project

- Work as a team
- Start *asap* with a team meeting
- Keep communicating *regularly* during semester
- Be fair to your team members
- Being able to work as a team is part of the exercise
- Be a team player

41

Research Project: Possible Failures

- **Don't do this:**
  - never meet
  - not respond to emails
  - "I don't have time right to work on this project in the next few months, why don't you start and I catch up later"
  - "I have a paper deadline in 1 month, cannot do anything else right now"
  - **while** not desparate(project-partners) **do**
    "I do my part until end of next week"
    … nothing happens …
    end
  - "why don't you take care of the presentation"
  - "why don't you take care of the report, I'll do the project presentation"
- **Single point of failure:**
  - One team member is the expert on the project and says: I quickly code up the basic infrastructure, then the three of you can join working on parts
  - 1 month later, the "quickly coding up" …

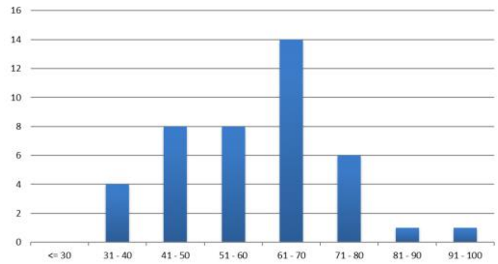
42

Midterm Exam

- **Covers first part of course**
- **Will fix time soon**
- ***No substitute date***

- ***There is no final exam***

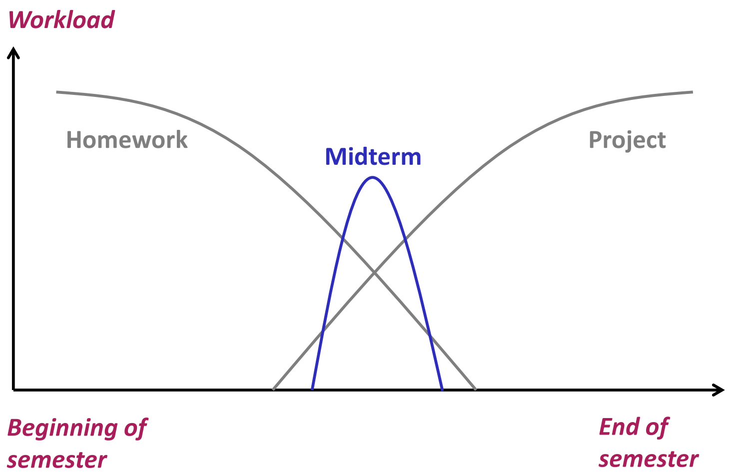
Point distribution 2016



Homework

- **Done individually**
- ***Solving homeworks analogous to homeworks in prior years is no guarantee for full points***
- **Exercises on algorithm/performance analysis**
- **Implementation exercises**
  - Concrete numerical problems
  - Study the effect of program optimizations, use of compilers, use of special instructions, etc. (Writing C code + creating runtime/performance plots)
  - Some templates will be provided
- **Homework is scheduled to leave time for research project**
- **Small part of homework grade for neatness**
- **Late homework policy:**
  - ***No deadline extensions***, but
  - 3 late days for the entire semester (at most 2 for one homework)

Workload During Semester (Sketch)



Academic Integrity

- **Zero tolerance cheating policy (cheat = fail + being reported)**
- **Homeworks**
  - All single-student
  - Don't look at other students code
  - Don't copy code from anywhere
  - Ok to discuss things – but then you have to do it alone
- **Code may be checked with tools**
- ***Don't do copy-paste***
  - code
  - ANY text
  - pictures
  - especially not from Wikipedia

Background Material

- See course website
- Prior versions of this course: see website
- I post all slides, notes, etc. on the course website

47

Class Participation

- I'll start on time
- It is important to attend
  - Most things I'll teach are not in books
  - I'll use part slides part blackboard
- Do ask questions
- We like when people come to office hours

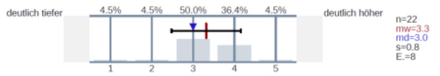
- *If you drop the course, please unregister from edoz*

48

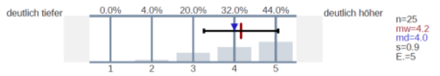
From the Course Evaluation 2017

Master students CS

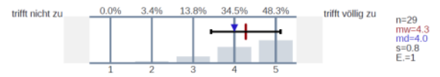
Im Vergleich zu anderen Lehrveranstaltungen derselben Kurskategorie war der fachliche Anspruch...



Im Vergleich zu anderen Lehrveranstaltungen derselben Kurskategorie war der Aufwand pro Kreditpunkt...



Die Vorlesung hat mich begeistert und für das Gebiet motiviert.



Student Numbers

- **Limited to 84**
- **Right now: 32 on wait list (similar to last time)**
- **Last time in the end: 60**
- **Last date for deregistering: March 17th**