How to Write Fast Numerical Code

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Lecture: Roofline model

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Operational Intensity Again

- **Definition: Given a program P, assume cold (empty) cache**

Operational intensity: $I(n) = \frac{W(n)}{Q(n)}$

- $W(n)$: #flops (input size n)
- $Q(n)$: #bytes transferred cache ↔ memory (for input size n)

- **Examples: Determine asymptotic bounds on I(n)**
  - Vector sum: y = x + y — **O(1)**
  - Matrix-vector product: y = Ax — **O(1)**
  - Fast Fourier transform — **O(log(n))**
  - Matrix-matrix product: C = AB + C — **O(n)**

Compute/Memory Bound

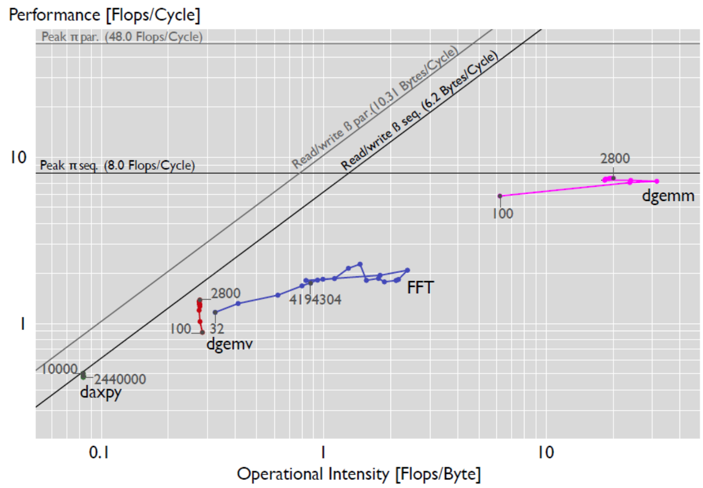
- **A function/piece of code is:**
  - ***Compute bound*** if it has high operational intensity
  - ***Memory bound*** if it has low operational intensity
- **The roofline model makes this more precise**
- **Blackboard**

Roofline Measurements

- **Tool developed in our group**
  (G. Ofenbeck, R. Steinmann, V. Caparros-Cabezas, D. Spampinato)
  http://www.spiral.net/software/roofline.html
- **You can use it in your project**
- **Example plots follow**
- **Get (non-asymptotic) bounds on I:**
  - daxpy: $y = \alpha x+y$
  - dgemv: $y = Ax + y$
  - dgemm: $C = AB + C$
  - FFT

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Intel MKL, ***sequential***
Cold cache

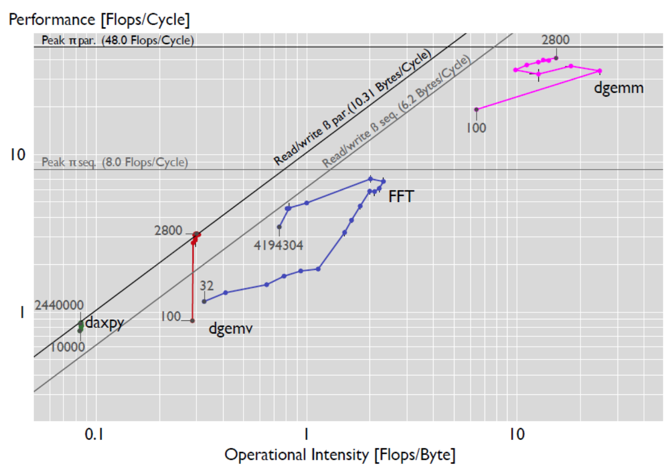


What happens when we go to parallel code?

5

Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Intel MKL, ***parallel***
Cold cache

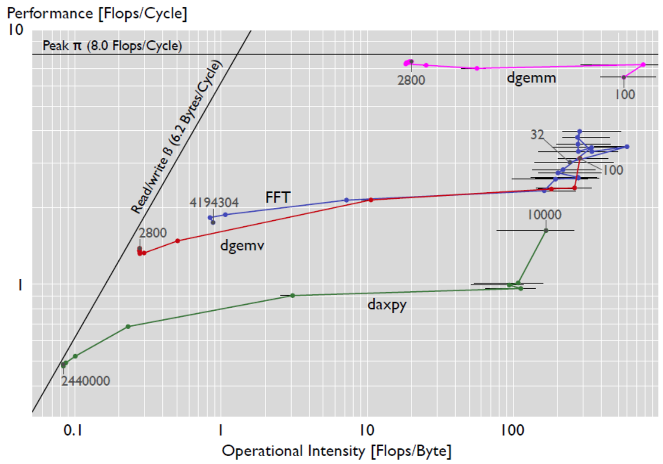


What happens when we go to warm cache?

6

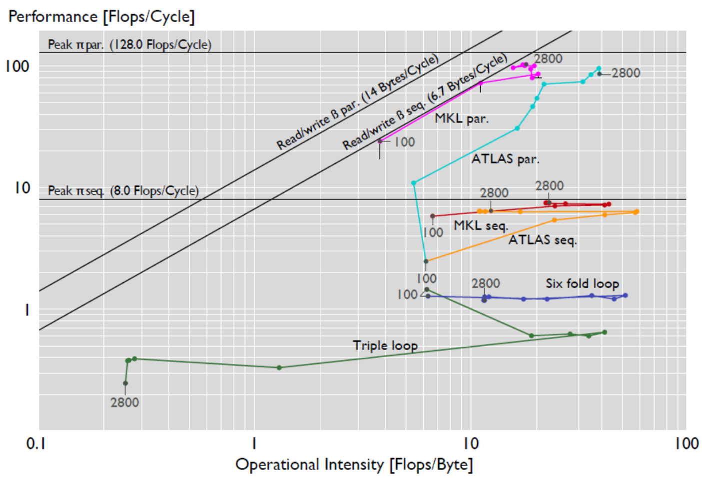
Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Intel MKL, ***sequential***
Warm cache



Roofline Measurements

Core i7 Sandy Bridge, 6 cores
Code: Various MMM
Cold cache



MMM: Try to guess the basic shapes

Summary

- Roofline plots distinguish between memory and compute bound
- Can be used on paper
- Measurements difficult (performance counters) but doable
- Interesting insights: *use in your project!*