How to Write Fast Numerical Code

Spring 2015

Lecture: Optimization for Instruction-Level Parallelism

Instructor: Markus Püschel

TA: Gagandeep Singh, Daniele Spampinato, Alen Stojanov

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Organizational

- **Midterm:** ***April 15th***
- **Office hours fixed**
- **Projects**

How To Make Code Faster?

- **It depends!**
- **Memory bound: Reduce memory traffic**
  - Reduce cache misses, register spills
  - Compress data
- **Compute bound: Keep floating point units busy**
  - Reduce cache misses, register spills
  - Instruction level parallelism (ILP)
  - Vectorization
- **Next: Optimizing for ILP (an example)**

Chapter 5 in ***Computer Systems: A Programmer's Perspective****, 2nd edition, Randal E. Bryant and David R. O'Hallaron, Addison Wesley 2010*
Part of these slides are adapted from the course associated with this book

3

Superscalar Processor

- **Definition: A superscalar processor can issue and execute *multiple instructions in one cycle*. The instructions are retrieved from a sequential instruction stream and are usually scheduled dynamically.**
- **Benefit: Superscalar processors can take advantage of *instruction level parallelism (ILP)* that many programs have**
- **Most CPUs since about 1998 are superscalar**
- **Intel: since Pentium Pro**

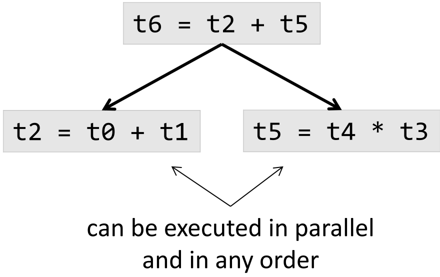
4

ILP

Code

```
t2 = t0 + t1
t5 = t4 * t3
t6 = t2 + t5
```

Dependencies



Hard Bounds: Pentium 4 vs. Core 2

- **Pentium 4 (Nocona)**

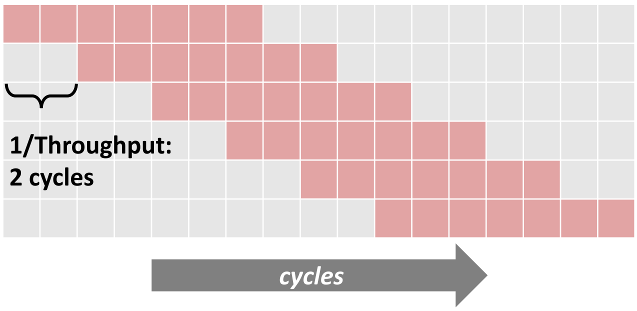
Instruction	*Latency*	*1/Throughput = Cycles/Issue*
Load / Store	5	1
Integer Multiply	10	1
Integer/Long Divide	36/106	36/106
Single/Double FP Multiply	**7**	**2**
Single/Double FP Add	**5**	**2**
Single/Double FP Divide	32/46	32/46

put on black-board (FP Multiply and FP Add rows)

- **Core 2**

Instruction	*Latency*	*Cycles/Issue*
Load / Store	5	1
Integer Multiply	3	1
Integer/Long Divide	18/50	18/50
Single/Double FP Multiply	**4/5**	**1**
Single/Double FP Add	**3**	**1**
Single/Double FP Divide	18/32	18/32

Single/Double FP Multiply	**7**	**2**



Hard Bounds (cont'd)

- **How many cycles at least if**
  - Function requires n float adds?
  - Function requires n int mults?

Example Computation (on Pentium 4)

```
void combine4(vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(v);
  data_t *d  = get_vec_start(v);
  data_t t   = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

d[0] OP d[1] OP d[2] OP … OP d[length-1]

data_t: float or double or int

OP: + or *

IDENT: 0 or 1

Runtime of Combine4 (Pentium 4)

- **Use cycles/OP**

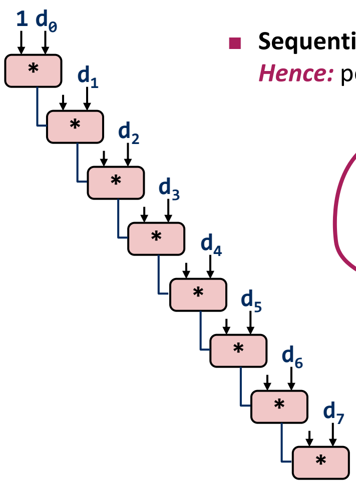
```
void combine4(vec_ptr v, data_t *dest)
{
  int i;
  int length = vec_length(v);
  data_t *d  = get_vec_start(v);
  data_t t   = IDENT;
  for (i = 0; i < length; i++)
    t = t OP d[i];
  *dest = t;
}
```

- **Questions:**
  - Explain red row
  - Explain gray row

Cycles per OP

Method	Int (add/mult)		Float (add/mult)	
combine4	2.2	10.0	5.0	7.0
bound	1.0	1.0	2.0	2.0

Combine4 = Serial Computation (OP = *)



- **Sequential dependence = no ILP!**
  Hence: performance determined by latency of OP!

Cycles per element (or per OP)

Method	Int (add/mult)		Float (add/mult)	
combine4	2.2	10.0	5.0	7.0
bound	1.0	1.0	2.0	2.0

Loop Unrolling

```
void unroll2(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit  = length-1;
    data_t *d  = get_vec_start(v);
    data_t x   = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i += 2)
        x = (x OP d[i]) OP d[i+1];
    /* Finish any remaining elements */
    for (; i < length; i++)
        x = x OP d[i];
    *dest = x;
}
```

- **Perform 2x more useful work per iteration**

Effect of Loop Unrolling

Method	Int (add/mult)		Float (add/mult)	
combine4	2.2	10.0	5.0	7.0
unroll2	1.5	10.0	5.0	7.0
bound	1.0	1.0	2.0	2.0

- **Helps integer sum**
- **Others don't improve. *Why?***
  - Still sequential dependency

```
x = (x OP d[i]) OP d[i+1];
```


Loop Unrolling with Reassociation

```
void unroll2_ra(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit  = length-1;
    data_t *d  = get_vec_start(v);
    data_t x   = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i += 2)
        x = x OP (d[i] OP d[i+1]);
    /* Finish any remaining elements */
    for (; i < length; i++)
        x = x OP d[i];
    *dest = x;
}
```

- **Can this change the result of the computation?**
- **Yes, for FP. *Why?***

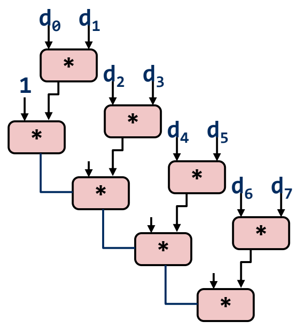
Effect of Reassociation

Method	Int (add/mult)		Float (add/mult)	
combine4	2.2	10.0	5.0	7.0
unroll2	1.5	10.0	5.0	7.0
unroll2-ra	1.56	5.0	2.75	3.62
bound	1.0	1.0	2.0	2.0

- **Nearly 2x speedup for Int *, FP +, FP ***
  - Why is that? (next slide)

Reassociated Computation

```
x = x OP (d[i] OP d[i+1]);
```



- **Breaks sequential dependency**
- **Overall Performance**
  - N elements, D cycles latency/op
  - Should be (N/2+1)*D cycles:
    cycle per OP ≈ D/2
  - Measured is slightly worse for FP

Loop Unrolling with Separate Accumulators

```
void unroll2_sa(vec_ptr v, data_t *dest)
{
    int length = vec_length(v);
    int limit  = length-1;
    data_t *d  = get_vec_start(v);
    data_t x0  = IDENT;
    data_t x1  = IDENT;
    int i;
    /* Combine 2 elements at a time */
    for (i = 0; i < limit; i+=2) {
       x0 = x0 OP d[i];
       x1 = x1 OP d[i+1];
    }
    /* Finish any remaining elements */
    for (; i < length; i++)
        x0 = x0 OP d[i];
    *dest = x0 OP x1;
}
```

- **Different form of reassociation**

Effect of Separate Accumulators

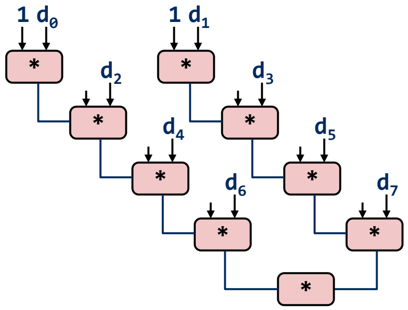
Method	Int (add/mult)		Float (add/mult)	
combine4	2.2	10.0	5.0	7.0
unroll2	1.5	10.0	5.0	7.0
unroll2-ra	1.56	5.0	2.75	3.62
unroll2-sa	1.50	5.0	2.5	3.5
bound	1.0	1.0	2.0	2.0

- **Almost exact 2x speedup (over unroll2) for Int *, FP +, FP ***
  - Breaks sequential dependency in a “cleaner,” more obvious way

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```

Separate Accumulators

```
x0 = x0 OP d[i];
x1 = x1 OP d[i+1];
```



- **What changed:**
  - Two independent "streams" of operations
- **Overall Performance**
  - N elements, D cycles latency/op
  - Should be (N/2+1)*D cycles: ***cycles per OP ≈ D/2***

What Now?

Unrolling & Accumulating

- **Idea**
  - Use K accumulators
  - Increase K until best performance reached
  - Need to unroll by L, K divides L
- **Limitations**
  - Diminishing returns:
    Cannot go beyond throughput limitations of execution units
  - Large overhead for short lengths: Finish off iterations sequentially

Unrolling & Accumulating: Intel FP *

- **Case**
  - Pentium 4
  - FP Multiplication
  - Theoretical Limit: 2.00

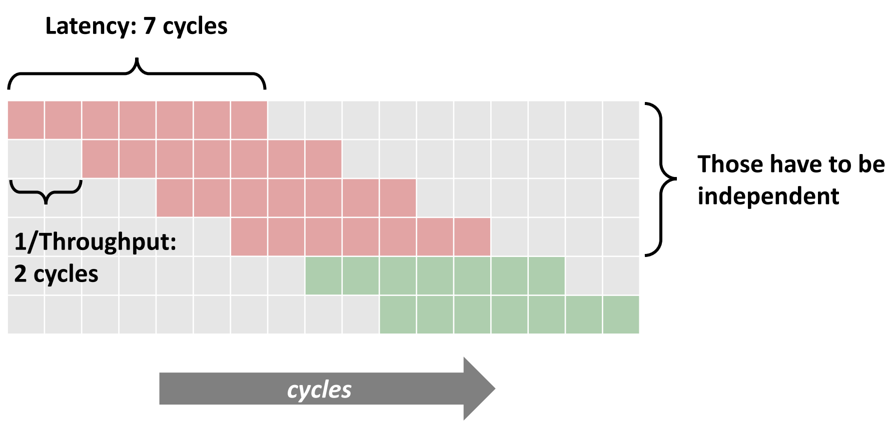
Accumulators

FP *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	7.00	7.00		7.01		7.00		
2		3.50		3.50		3.50		
3			2.34					
4				2.01		2.00		
6					2.00			2.01
8						2.01		
10							2.00	
12								2.00

Why 4?

21

Why 4?



Based on this insight: K = #accumulators = ceil(latency/cycles per issue)

22

Unrolling & Accumulating: Intel FP +

- **Case**
  - Pentium 4
  - FP Addition
  - Theoretical Limit: 2.00

FP +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	5.0	5.0		5.0		5.0		
2		2.5		2.5		2.5		
3			2.0					
4				2.0		2.00		
6					2.0			2.0
8						2.0		
10							2.0	
12								2.0

Unrolling & Accumulating: Intel Int *

- **Case**
  - Pentium 4
  - Integer Multiplication
  - Theoretical Limit: 1.00

Int *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	10.0	10.0		10.0		10.0		
2		5.0		5.0		5.0		
3			3.3					
4				2.5		2.5		
6					1.67			1.67
8						1.25		
10							1.1	
12								1.14

Unrolling & Accumulating: Intel Int +

- **Case**
  - Pentium 4
  - Integer addition
  - Theoretical Limit: 1.00

Int +	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	2.2	1.5		1.1		1.0		
2		1.5		1.1		1.0		
3			1.34					
4				1.1		1.03		
6					1.0			1.0
8						1.03		
10							1.04	
12								1.1

FP *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	7.0	7.0		7.0		7.0		
2		3.5		3.5		3.5		
3			2.34					
4				2.0		2.0		
6					2.0			2.0
8						2.0		
10							2.0	
12								2.0

Pentium 4

FP *	Unrolling Factor L							
K	1	2	3	4	6	8	10	12
1	4.0	4.0		4.0		4.0		
2		2.0		2.0		2.0		
3			1.34					
4				1.0		1.0		
6					1.0			1.0
8						1.0		
10							1.0	
12								1.0

Core 2
*FP * is fully pipelined*

Summary (ILP)

- **Instruction level parallelism may have to be made explicit in program**
- **Potential blockers for compilers**
  - Reassociation changes result (FP)
  - Too many choices, no good way of deciding
- **Unrolling**
  - By itself does often nothing (branch prediction works usually well)
  - But may be needed to enable additional transformations (here: reassociation)
- **How to program this example?**
  - Solution 1: program generator generates alternatives and picks best
  - Solution 2: use model based on latency and throughput