How to Write Fast Numerical Code

Spring 2015

Lecture: Architecture/Microarchitecture and Intel Core

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Technicalities

- **Midterm: *April 15th* (during recitation time)**
- **Research project:**
  - Let us know once you have a partner
  - If you have a project idea, talk to me (break, after Wed class, email)
  - Deadline: March 6th
- **Finding partner: fastcode-forum@lists.inf.ethz.ch**
  - Recipients: TA + all students that have no partner yet
- **We will be using Moodle for homeworks (online submission and more)**

Today

- Architecture/Microarchitecture: *What is the difference?*
- In detail: Core 2/Core i7
- Crucial microarchitectural parameters
- Peak performance
- Operational intensity

Definitions

- *Architecture (also instruction set architecture = ISA):* The parts of a processor design that one needs to understand to write assembly code
- *Examples:* instruction set specification, registers
- *Counterexamples:* cache sizes and core frequency
- Example ISAs
  - x86
  - ia
  - MIPS
  - POWER
  - SPARC
  - ARM

Some assembly code

```
ipf:
    xorps   %xmm1, %xmm1
    xorl    %ecx, %ecx
    jmp     .L8
.L10:
    movslq  %ecx,%rax
    incl    %ecx
    movss (%rsi,%rax,4), %xmm0
    mulss (%rdi,%rax,4), %xmm0
    addss   %xmm0, %xmm1
.L8:
    cmpl    %edx, %ecx
    jl      .L10
    movaps  %xmm1, %xmm0
    ret
```

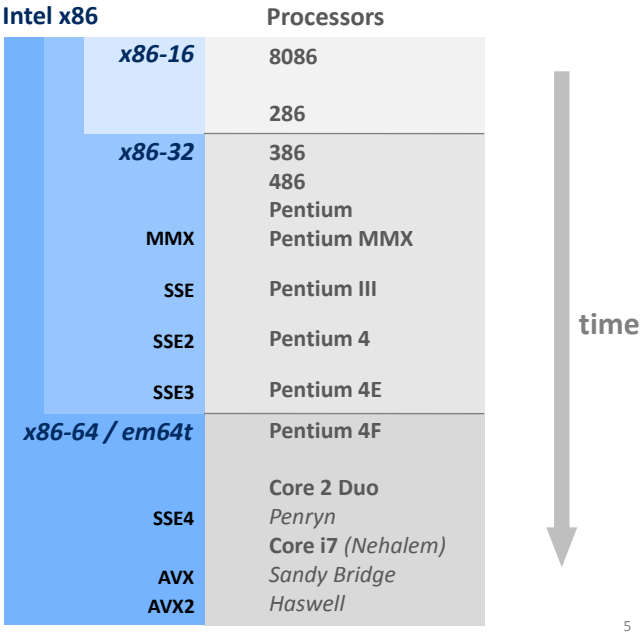
MMX:
Multimedia extension

SSE:
Streaming SIMD extension

AVX:
Advanced vector extensions

Backward compatible:
Old binary code (≥ 8086) runs on new processors.

New code to run on old processors?
Depends on compiler flags.

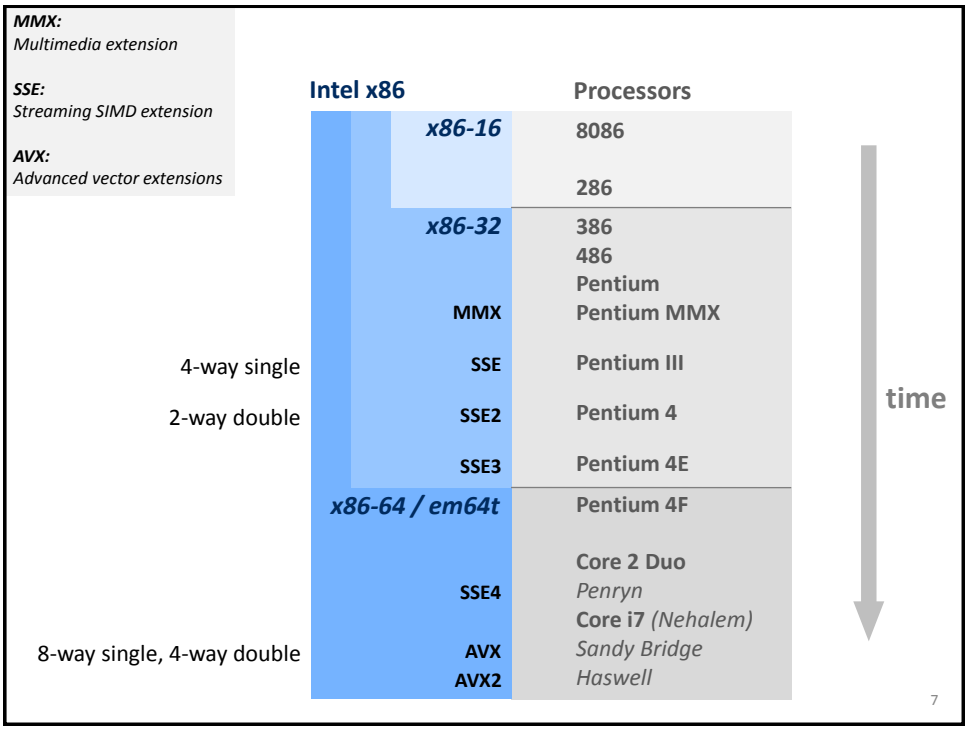


ISA SIMD (Single Instruction Multiple Data) Vector Extensions

- **What is it?**
  - Extension of the ISA. Data types and instructions for the parallel computation on short (length 2-8) vectors of integers or floats

4-way

  - Names: MMX, SSE, SSE2, …, AVX, …
- **Why do they exist?**
  - ***Useful:*** Many applications have the necessary fine-grain parallelism Then: speedup by a factor close to vector length
  - ***Doable:*** Chip designers have enough transistors to play with
- **We will have an extra lecture on vector instructions**
  - What are the problems?
  - How to use them efficiently

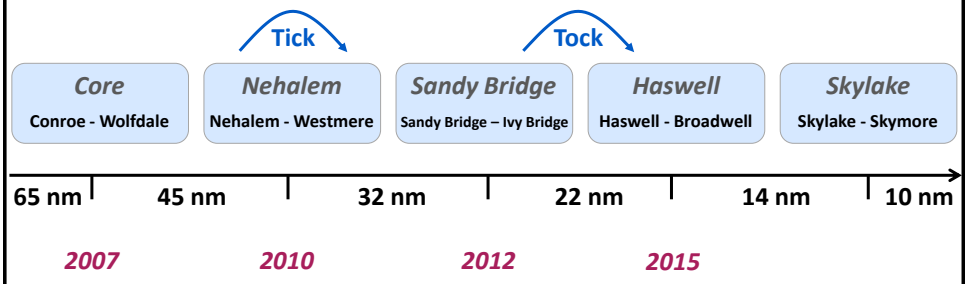


Definitions

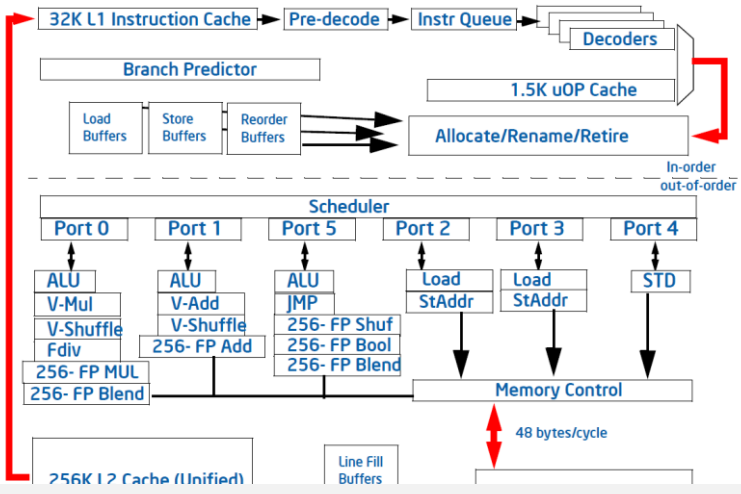
- *Microarchitecture:* **Implementation of the architecture**
- *Examples:* **caches, cache structure, CPU frequency, details of the virtual memory system**
- **Examples**
  - Intel processors (Wikipedia)
  - AMD processors (Wikipedia)

Intel's Tick-Tock Model

- **Tick: Shrink of process technology**
- **Tock: New microarchitecture**
- **Example: Core and successors**
  Shown: Intel's microarchitecture code names (server/mobile may be different)



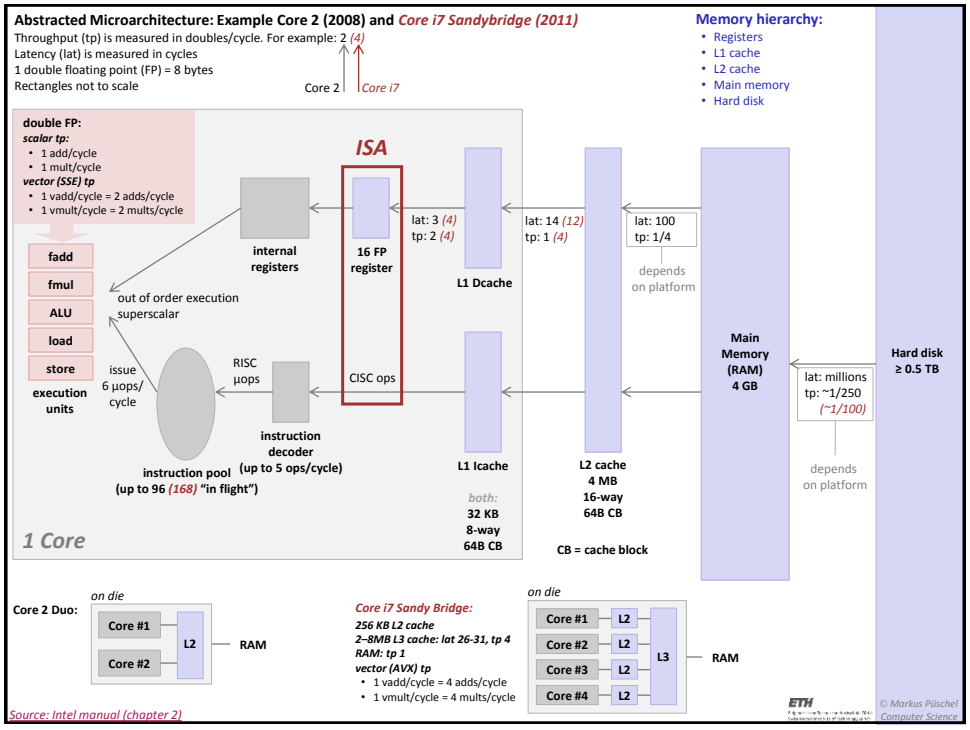
Microarchitecture: The View of the Computer Architect



we take the software developer's view …

Source: Intel Architectures Optimization Reference Manual

- **Distribute microarchitecture abstraction**



Runtime Bounds (Cycles) on Core 2

```
/* dot product computation; x, y are vectors of doubles of length n */
double t = 0;
for (i = 0; i < n; i++)
  t = t + x[i]*y[i];
```

maximal achievable percentage of (vector) peak performance

■ Number flops?	2n	
■ Runtime bound no vector ops:	n	
■ Runtime bound vector ops:	n/2	
■ Runtime bound data in L1:	n	50
■ Runtime bound data in L2:	2n	25
■ Runtime bound data in main memory:	8n	6.25

Runtime dominated by data movement: Memory-bound

Runtime Bounds (Cycles) on Core 2

```
/* matrix multiplication; A, B, C are n x n matrices of doubles */
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      C[i*n+j] += A[i*n+k]*B[k*n+j];
```

■ Number flops?	$2n^3$
■ Runtime bound no vector ops:	n^3
■ Runtime bound vector ops:	$n^3/2$
■ Runtime bound data in L1:	$3/2\ n^2$
■ Runtime bound data in L2:	$3n^2$
■ Runtime bound data in main memory:	$12n^2$

Runtime dominated by data operations (except very small n): Compute-bound

Operational Intensity

- **Definition: Given a program P, assume cold (empty) cache**

$$\textit{Operational intensity: } I(n) = \frac{W(n)}{Q(n)}$$

W(n): #flops (input size n)

Q(n): #bytes transferred cache ↔ memory (for input size n)

Operational Intensity (Cold Cache)

```
/* dot product computation; x, y are vectors of doubles of length n */
double t = 0;
for (i = 0; i < n; i++)
  t = t + x[i]*y[i];
```

- **Operational intensity:**
  - Flops: W(n) **= 2n**
  - Memory/cache transfers (doubles): **≥ 2n (just from the reads)**
  - Reads (bytes): Q(n) **≥ 16n**
  - Operational intensity: I(n) **= W(n)/Q(n) ≤ 1/8**

Operational Intensity (Cold Cache)

```
/* matrix multiplication; A, B, C are n x n matrices of doubles */
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      C[i*n+j] += A[i*n+k]*B[k*n+j];
```

- **Operational intensity:**
  - Flops: W(n) **$= 2n^3$**
  - Memory/cache transfers (doubles): **$\geq 3n^2$ (just from the reads)**
  - Reads (bytes): Q(n) **$\geq 24n^2$**
  - Operational intensity: I(n) **$= W(n)/Q(n) \leq 1/12\ n$**

Operational Intensity

- **Definition: Given a program P, assume cold (empty) cache**

$$\textit{Operational intensity: } I(n) = \frac{W(n)}{Q(n)}$$

W(n): #flops (input size n)

Q(n): #bytes transferred cache ↔ memory (for input size n)

- **Examples: Determine asymptotic bounds on I(n)**
  - Vector sum: y = x + y — **O(1)**
  - Matrix-vector product: y = Ax — **O(1)**
  - Fast Fourier transform — **O(log(n))**
  - Matrix-matrix product: C = AB + C — **O(n)**

- **Note:** In the last two cases, the tightest possible bound depends on the cache size m; more later

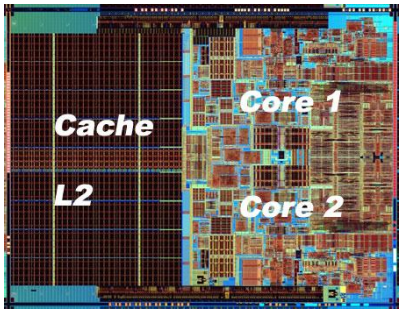
Compute/Memory Bound

- **A function/piece of code is:**
  - ***Compute bound*** if it has high operational intensity
  - ***Memory bound*** if it has low operational intensity
- **A more exact definition depends on the given platform**
- **More details later: Roofline model**

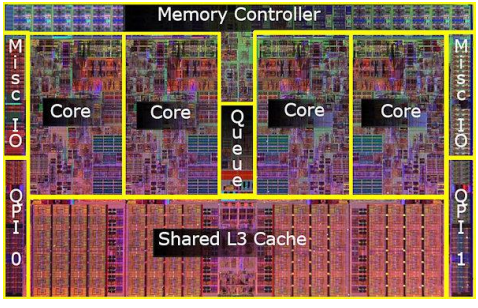
Pictures: Intel

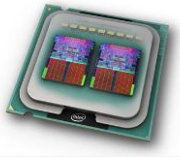
Core Processor

Core 2 Duo



Core i7





2 x Core 2 Duo packaged

Detailed information about Core processors

Floating Point Peak Performance?

Table 2-15. Issue Ports of Intel Core Microarchitecture and Enhanced Intel Core Microarchitecture

Executable operations	Latency, Throughput		Comment[1]
	Signature = 06_0FH	Signature = 06_17H	
Integer ALU Integer SIMD ALU FP/SIMD/SSE2 Move and Logic	1, 1 1, 1 1, 1	1, 1 1, 1 1, 1	Includes 64-bit mode integer MUL; Issue port 0; Writeback port 0;
Single-precision (SP) FP MUL Double-precision FP MUL	4, 1 5, 1	4, 1 5, 1	Issue port 0; Writeback port 0
FP MUL (X87) FP Shuffle DIV/SQRT	5, 2 1, 1	5, 2 1, 1	Issue port 0; Writeback port 0 FP shuffle does not handle QW shuffle.
Integer ALU Integer SIMD ALU FP/SIMD/SSE2 Move and Logic	1, 1 1, 1 1, 1	1, 1 1, 1 1, 1	Excludes 64-bit mode integer MUL; Issue port 1; Writeback port 1;
FP ADD QW Shuffle	3, 1 1, 1[2]	3, 1 1, 1[3]	Issue port 1; Writeback port 1;
Integer loads FP loads	3, 1 4, 1	3, 1 4, 1	Issue port 2; Writeback port 2;
Store address[4]	3, 1	3, 1	Issue port 3;
Store data[5].			Issue Port 4;
Integer ALU Integer SIMD ALU FP/SIMD/SSE2 Move and Logic	1, 1 1, 1 1, 1	1, 1 1, 1 1, 1	Issue port 5; Writeback port 5;
QW shuffles 128-bit Shuffle/Pack/Unpack	1, 1[2] 2-4, 2-4[6]	1, 1[3] 1-3, 1[7]	Issue port 5; Writeback port 5;

Two different processor lines

1 or 2 mults/cycle?

1 add/cycle

The Two Floating Points

```
float ipf (float x[], float y[], int n) {
  int i;
  float result = 0.0;

  for (i = 0; i < n; i++)
    result += x[i]*y[i];
  return result;
}
```

standard compilation (SSE)

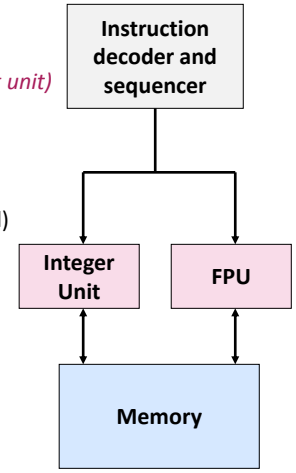
```
ipf:
   xorps   %xmm1, %xmm1
   xorl    %ecx, %ecx
   jmp     .L8
.L10:
   movslq  %ecx,%rax
   incl    %ecx
   movss (%rsi,%rax,4), %xmm0
   mulss (%rdi,%rax,4), %xmm0
   addss   %xmm0, %xmm1
.L8:
   cmpl    %edx, %ecx
   jl      .L10
   movaps  %xmm1, %xmm0
   ret
```

compilation for x87

```
…
cmpl %edx,%eax
   jge .L3
.L5:
   flds (%ebx,%eax,4)
   fmuls (%ecx,%eax,4)
   faddp
   incl %eax
   cmpl %edx,%eax
   jl .L5
.L3:
   movl -4(%ebp),%ebx
   movl %ebp, %esp
   popl %ebp
   ret
```

The Other Floating Point (x87)

- **History**
  - 8086: first computer to implement IEEE FP
    (separate 8087 math coprocessor with floating point unit)
  - Logically stack based
  - 486: merged FPU and Integer Unit onto one chip
  - Once SSE came out, it was used for floating point
  - x87 is default on x86-32 (since SSE is not guaranteed)
  - Became obsolete with x86-64



MMX:
Multimedia extension

SSE:
Streaming SIMD extension

AVX:
Advanced vector extensions

Intel x86	Processors
x86-16	8086
	286
x86-32	386
	486
	Pentium
MMX	Pentium MMX
SSE	Pentium III
SSE2	Pentium 4
SSE3	Pentium 4E
x86-64 / em64t	Pentium 4F
	Core 2 Duo
SSE4	*Penryn*
	Core i7 *(Nehalem)*
AVX	*Sandy Bridge*
AVX2	*Haswell*

time

The Other Floating Point (x87)

- **History**
  - 8086: first computer to implement IEEE FP
    (separate 8087 math coprocessor with floating point unit)
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  - Once SSE came out, it was used for floating point
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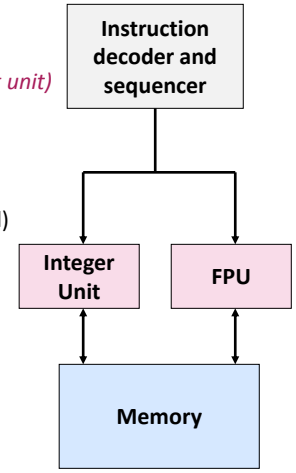
  - Became obsolete with x86-64
- **Floating Point Formats**
  - single precision (C **`float`**): 32 bits
  - double precision (C **`double`**): 64 bits
  - extended precision (C **`long double`**): 80 bits



Core: Floating Point Peak Performance

Single-precision (SP) FP MUL Double-precision FP MUL	4, 1 5, 1	4, 1 5, 1	Issue port 0; Writeback port 0
FP MUL (X87) FP Shuffle DIV/SQRT	5, 2 1, 1	5, 2 1, 1	Issue port 0; Writeback port 0 FP shuffle does not handle QW shuffle.

SSE based FP
x87 FP

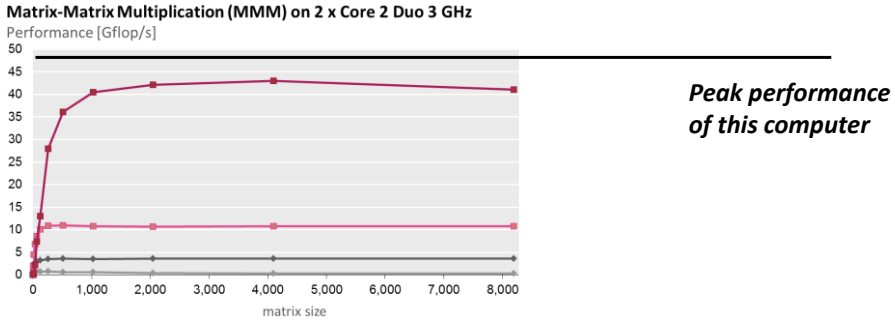
- **Scalar:**
  - 1 add and 1 mult / cycle: 2 flops/cycle
  - Assume 3 GHz:
    6 Gflop/s scalar peak performance on one core
- **Vector double precision (SSE2)**
  - 1 vadd and 1 vmult / cycle (2-way): 4 flops/cycle
  - Assume 3 GHz:
    12 Gflop/s peak performance on one core
- **Vector single precision (SSE)**
  - 1 vadd and 1 vmult / cycle (4-way): 8 flops/cycle
  - Assume 3 GHz:
    24 Gflop/s peak performance on one core

Core: Floating Point Peak Performance

- **Overall peak on 3 GHz Core 2 and Core i3/i5/i7 Nehalem: (2 cores, SSE)**
  - Double precision: ***24 Gflop/s***
  - Single precision: ***48 Gflop/s***

- **Overall peak on 3 GHz Core i3/i5/i7 Sandy Bridge: (4 cores, AVX)**
  - Double precision: ***96 Gflop/s***
  - Single precision: ***192 Gflop/s***

Example: Peak Performance



Peak performance of this computer

Summary

- **Architecture vs. microarchitecture**
- **To optimize code one needs to understand a suitable abstraction of the microarchitecture**
- **Operational intensity:**
  - High = compute bound = runtime dominated by data operations
  - Low = memory bound = runtime dominated by data movement