Fourier Analysis-based Iterative Combinatorial Auctions

Jakob Weissteiner*, Chris Wendler*, Sven Seuken, Ben Lubin, Markus Püschel

*equal contribution

**Combinatorial Auctions**

**Goal**  efficiently allocate items to bidders

**Challenge**  combinatorial explosion of bundles

**Example**  spectrum auction

- 17 spectrum licenses
- $2^{17} = 131,072$ values ($2^m$ in general)

**Solution**

Control this complexity using recent Fourier analysis [Püschel and Wendler, 2020] of set functions

- Fourier reconstruction queries
- Fourier-sparse bidder representation
- Efficient winner determination problem

**Set Function Fourier Analysis**

- **Spectral Energy Distribution in LSVM (2 bidder types, m = 18)**
- **Value function** $v : \{0, 1\}^m \rightarrow \mathbb{R}$
- $v(x) = \sum_y F_{xy}^{-1} \phi_y(y)$
- Fourier coefficients (FCs)

Sparse Fourier transform

- $k \ll 2^m$
- computes $\tilde{v}(x) = \sum_{y=0}^{\delta} F_{xy}^{-1} \phi_y(y)$
- with small $\|\tilde{v} - v\|^2$ from value queries

**Sparse Fourier Transform as Preference Elicitation**

**Bidder 1**

$v_1(a_1)$

$R_1^{-1}$

Reports

$\tilde{v}_1(a_1)$

**Bidder 2**

$v_2(a_2)$

$R_2^{-1}$

Reports

$\tilde{v}_2(a_2)$

**Bidder n**

$v_n(a_n)$

$R_n^{-1}$

Reports

$\tilde{v}_n(a_n)$

Query values for $\alpha^* = (a_1^*, ..., a_n^*)$

100% economic efficiency

193,000 value queries per bidder

**Hybrid Iterative Combinatorial Auction**

**Idea**

query neural network (NN) instead of bidders & leverage compressive sensing [Stobbe and Krause, 2012]

**Estimation Step**

1. Fit NN to current queries
2. Compute Sparse Fourier transform of NN
3. Use 2. to define compressive sensing problem

**Optimization Step**

$\alpha^* = \text{argmax } \sum_{i} \tilde{v}_i(a_i)$

**NN's and true Fourier coefficients overlap**

**Experimental Evaluation**

Simulate spectrum auctions using the spectrum auction test suite (SATS) [Weiss et al., 2017]

- Global synergy value model (GSVM) [Goeree and Holt, 2010]
- 18 items, 6 regional and 1 national bidder

**GSVM**

<table>
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<tr>
<th>Mechanism</th>
<th>Efficiency</th>
<th>Regional</th>
<th>National</th>
<th>REV</th>
<th>BRB/EN</th>
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**Local Synergy value model (LSVM) [Scheffel et al., 2012]**

- 18 items, 5 regional and 1 national bidder
- Complementarities arise from spatial proximity

**LSVM**

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**Multi-region value model (MRVM) [Weiss et al., 2017]**

- 98 items and 10 bidders (local, regional and national)
- Models large US spectrum auctions

**MRVM**

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**Check out our Implementation**

- Calculate Fourier spectra of your own set functions
- $\texttt{python analyze_spectral_energy.py}$

or run Hybrid ICA to find efficient allocations!

[GitHub.com/marketdesignresearch/FA-based-ICAs]