Fourier Analysis-based Iterative Combinatorial Auctions

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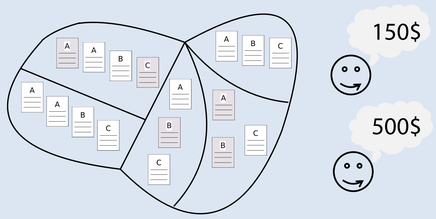
Combinatorial Auctions

Goal efficiently allocate m items to n bidders

Challenge combinatorial explosion of bundles

Example spectrum auction



17 spectrum licenses

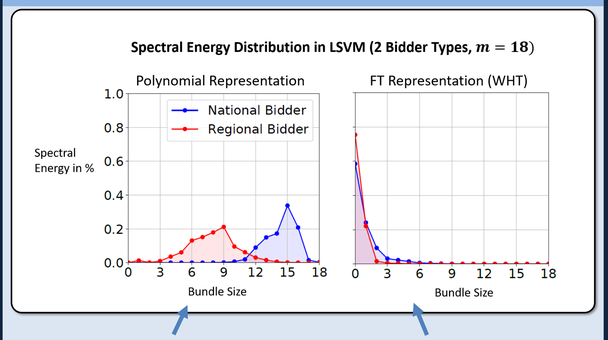
$2^{17} = 131.072$ values (2^m in general)

Solution

Control this **complexity** using recent **Fourier analysis [Püschel and Wendler, 2020]** of set functions

- Fourier reconstuction **queries**
- Fourier-sparse **bidder representation**
- **Efficient** winner determination problem

Set Function Fourier Analysis



value function

spectrum

$v : \{0,1\}^m \to \mathbb{R}$

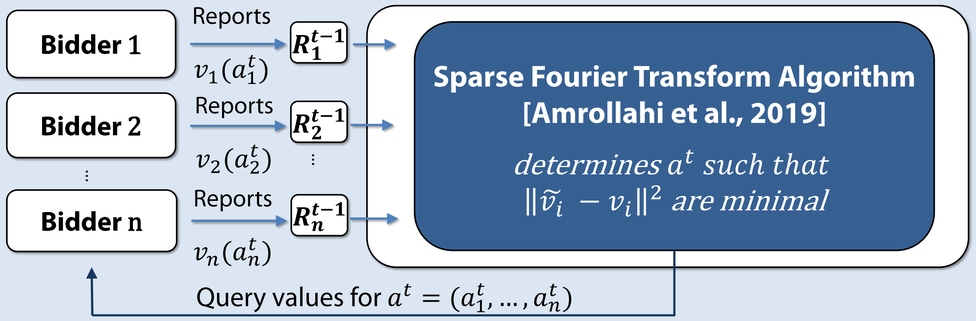
$v(x) = \sum_y F^{-1}_{xy} \phi_v(y)$ Fourier coefficients (FCs)

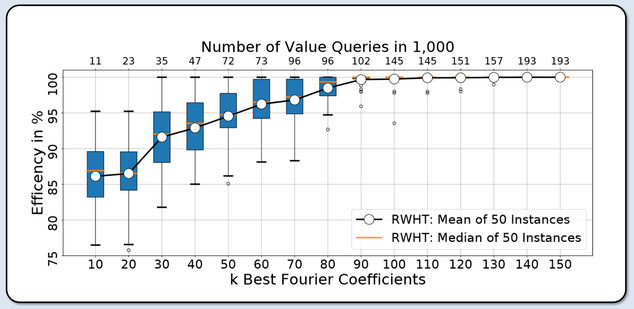
Sparse Fourier transform $k \ll 2^m$

computes $\tilde{v}(x) = \sum_{i=1}^{k} F^{-1}_{xy_i} \phi_{\tilde{v}}(y_i)$

with small $\|\tilde{v} - v\|^2$ from value queries

Sparse Fourier Transform as Preference Elicitation





100% economic efficiency ☺

193 000 value queries per bidder ☹

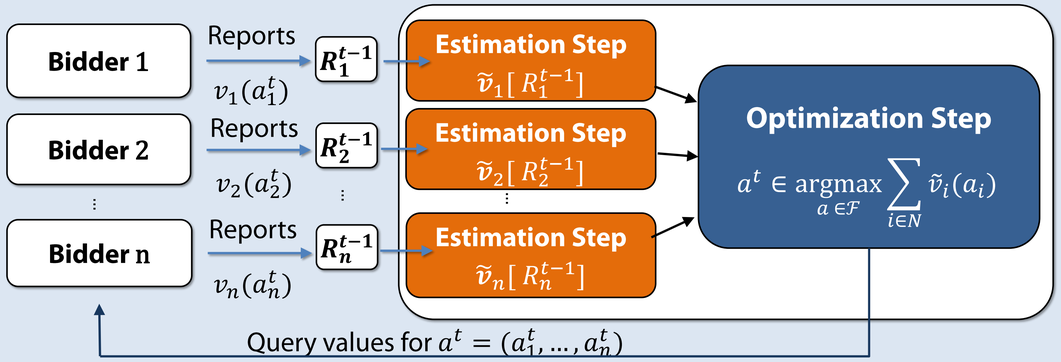
build a query efficient algorithm

Hybrid Iterative Combinatorial Auction

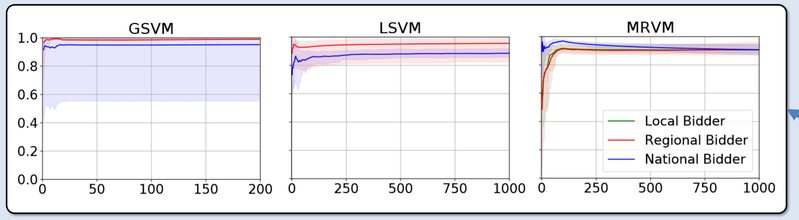
Idea

query **neural network (NN)** instead of bidders & leverage **compressive sensing [Stobbe and Krause, 2012]**

Estimation Step
1. Fit NN to current queries
2. Compute Sparse Fourier transform of NN
3. Use 2. to define compressive sensing problem



x-axis number of FCs
y-axis energy overlap



NN's and true Fourier coefficients overlap

Experimental Evaluation

Simulate spectrum auctions using the **spectrum auction test suite (SATS) [Weiss et al., 2017]**

Global synergy value model (GSVM) [Goeree and Holt, 2010]
18 items, 6 regional and 1 national bidder

	GSVM				
Mechanism	Efficiency in %	Regional in %	National in %	Rev in %	hrs/Inst.
Hybrid ICA	99.97 ± 0.03	94.72	5.25	81	0.81
MLCA	99.17 ± 0.37	98.11	1.06	79	4.65
Hybrid ICA (no FR)	98.30 ± 0.49	97.94	0.36	75	0.93
Hybrid ICA (no FR/FA)	98.16 ± 0.50	97.47	0.69	75	0.71
Efficient Allocation		94.75	5.25		

Local synergy value model (LSVM) [Scheffel et al., 2012]
18 items, 5 regional and 1 national bidder
complementarities arise from spatial proximity

	LSVM				
Mechanism	Efficiency in %	Regional in %	National in %	Rev in %	hrs/Inst.
Hybrid ICA	98.74 ± 0.43	89.09	9.65	78	1.95
MLCA	99.14 ± 0.42	93.40	5.75	77	6.09
Hybrid ICA (no FR)	98.16 ± 0.60	93.83	4.33	72	2.03
Hybrid ICA (no FR/FA)	97.75 ± 0.63	92.78	5.27	72	1.86
Efficient Allocation		84.03	15.97		

Multi-region value model (MRVM) [Weiss et al., 2017]
98 items and 10 bidders (local, regional and national)
models large US spectrum auctions

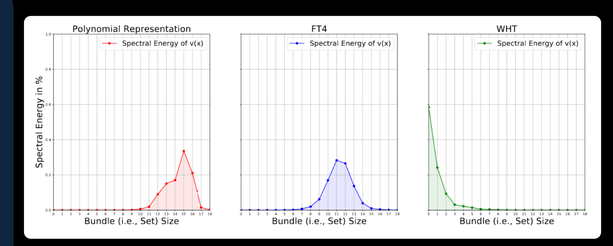
	MRVM					
Mechanism	Efficiency in %	Local in %	Regional in %	National in %	Rev in %	hrs/Inst.
Hybrid ICA	96.63 ± 0.31	0.00	1.19	95.44	36	23.88
MLCA	95.32 ± 0.32	0.00	0.53	94.79	41	43.26
Hybrid ICA (no FR)	96.63 ± 0.31	0.00	1.19	95.44	36	23.88
Hybrid ICA (no FR/FA)	93.91 ± 0.36	0.01	0.42	93.48	42	14.68
Efficient Allocation		0.00	2.11	97.89		

Hybrid ICA performs as good or better than state-of-the-art accross all SATS domains

Check out our Implementation

Calculate Fourier spectra of your own set functions

```
$ python analyze_spectral_energy.py
```



or run Hybrid ICA to find efficient allocations!

github.com/marketdesignresearch/FA-based-ICAs